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HEAT TRANSFER TO CLOUDS OF FALLING PARTICLES

BY

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AND

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION
BULLETIN SERIES No. 330

HEAT TRANSFER TO CLOUDS OF FALLING PARTICLES

BY

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HEAT TRANSFER TO CLOUDS OF FALLING PARTICLES

I. Introduction

1. *Object of Work.*—This investigation was undertaken for the purpose of obtaining information essential to the design of equipment for heating finely divided materials falling through a gas. The immediate object of the work was the design of a flash calciner for the decomposition of hydrated zinc sulphite, the product of one operation in a cyclic process for the recovery of sulphur dioxide from waste gases. A summary of the results and their application to that particular problem have been given in a previous bulletin.* Because of the general applicability of the work, the results are presented here in more detail along with the theoretical treatment essential to a complete development of the problem.

There is a growing interest in space reactions of solids, or between solids and gases, at high temperatures, because of the high rates of heat transfer to clouds of solid particles suspended in a gas. The method was suggested at one time for the carbonization of pulverized coal to produce "bubble coke."† Adherence of the particles to the retort walls, however, was not entirely prevented and the method did not gain favor, although it was recognized that the rate of heat transfer was rapid, and that conduction through the walls was actually a limiting factor on the capacity.‡ Application to the briquetting of blast furnace dust, the smelting of copper and zinc ores and for the production of soluble phosphates has been suggested.§ Other possible applications include the decomposition of sodium bicarbonate and the dehydration of clays and minerals. Several of these have been investigated in the experimental equipment to be described.

The general problem is also of interest in the design of spray dryers. Lapple and Shepherd¶ have recently made a theoretical study of the trajectories of the particles in such equipment. From the information given in their paper, it should be possible to apply the basic equations developed here for heat transfer to determine the heat removed from the particles and, thus, the point at which solidification, or crystallization is complete.

*Univ. of Ill. Eng. Exp. Sta. Bul. 324, 1940.

†A. H. White, Proc. First Internat. Conf. on Bit. Coal, Carnegie Inst., p. 419, 1926; W. Runge, *Ibid.*, p. 697; F. M. Gentry, Combustion, vol. 20, p. 225, 1929.

‡F. B. Hobart and D. J. Demorest, Ohio State Univ. Eng. Exp. Sta. Bul. 65, 1932.

§E. C. St. Jacques, Ind. Eng. Chem. News Ed., vol. 15, p. 29, 1937.

¶C. E. Lapple and C. B. Shepherd, Ind. Eng. Chem., vol. 32, p. 605, 1940.

2. *Acknowledgments.*—This work is a part of the research program of the Engineering Experiment Station, of which Dean M. L. Enger is the Director, and of the Division of Chemical Engineering, of which Professor D. B. Keyes is the Head. The research work was used as thesis subjects for advanced degrees in the Graduate School of the University by the junior authors. The clerical assistance of several students working under grants from the National Youth Administration is gratefully acknowledged.

II. Theoretical Discussion

3. *Nomenclature.*—Throughout this bulletin the equations are written in a form in which any consistent system of units may be used. In those cases in which designation of the units has been necessary, the English system has been used for quantities having immediate practical significance, and the metric system for quantities used principally in the laboratory. This policy, rather than one of strict uniformity, was adopted in order to secure greater clarity for the average reader.

a = average projected area of a single particle

A = surface area; subscript o refers to the total area emitting radiation, p to that of each particle, w to that of the internal wall of the furnace

b, B = arbitrary constants

c = specific heat of gas

C = concentration of particles at any point in the cloud, i.e., the number per unit volume

C_D = drag coefficient, defined by Equation (60)

d = differential operator

D = internal diameter of furnace tube

D_p = arithmetic average diameter of particles in the cloud

f = fraction of total area A covered by discs

F = feed rate, in general, expressed as lb./(min.) (sq. ft.)

F_{AE} = angle-emissivity factor for net radiation transfer

g = acceleration due to gravity

h = heat transfer coefficient; subscripts have the following meanings: m overall coefficient, r radiation coefficient, cp convection coefficient for spherical particle surface, cw convection coefficient at furnace wall. All coefficients *except* h_{cw} are based on particle area

I = intensity of radiation, subscript l refers to the intensity at point l measured from the front of the cloud, o refers to that at which $l = 0$, n refers to that in the direction normal to the surface

J_1 = Bessel function of the first order and the first kind

k = thermal conductivity

K = volume-shape factor, defined by Equation (59)

l = length of a radiant beam; subscript o refers to the total length of beam

L = length of path of particle, i.e., in general, the vertical distance along the furnace

M = a constant, defined by Equation (16)

N = number of particles present in the furnace at any instance

q = rate of heat transfer

r = radius; subscript l refers to radius of a spherical particle, subscript o refers to that of an infinite cylinder

Re = Reynolds number, $D_p V \rho_g / \mu$

s = thickness of hypothetical tube of gas

t = temperature; subscripts have the following meanings: g the temperature of the ambient gas, p that of the particle, s the uniform temperature of the surface to which heat is transferred, w the temperature of the furnace wall, ∞ the uniform temperature of the gas at infinite distance from the surface receiving heat

$\Delta_0 t = t_s - t_\infty$

T = absolute temperature, degrees Rankine; subscripts p and w refer to that of particle surface and wall surface, respectively

u, v, w = components of fluid velocity in the x, y, z directions, respectively

v_p = average volume of a particle

V = velocity of particles relative to furnace wall (in a stagnant gas); subscript t refers to terminal velocity

w_p = average mass of a particle

x, y, z = coordinates of a fluid element

$\alpha = \sqrt{\text{thermal diffusivity}} = \sqrt{k/c\rho}$

β = volume coefficient of thermal expansion

γ = ratio of particle area to wall area

Δ = temperature rise

ϵ_c = absorptivity of cloud of particles, i.e., fraction of radiation incident on cloud that is absorbed

ϵ_p = emissivity of particle surface

η = roots of first order Bessel function

λ = characteristic linear dimension
μ = viscosity of gas
π = ratio of circumference to diameter
Π, Ψ = conformal coordinates for potential flow around a submerged body of revolution. Note that $V\Pi$ is the velocity potential
ρ = density; subscript p and g refer to the particle and the gas, respectively
θ = time; $\Delta\theta_f$ = time of fall through furnace
φ = angular coordinate
ω = solid angle

4. *Mechanism of Heat Transfer.*—The transfer of heat from a solid surface through a gas to falling particles takes place both by convection and by radiation. The thermal resistance to convection is composed of the resistance from the wall to the gas and that from the gas to the particles, acting in series. Transfer of radiant energy takes place directly from the wall to the particles. If the gas has infra-red absorption bands, it also will receive heat directly from the wall and, consequently, it will partially blanket the particles from radiant energy. The net radiation transfer acts in parallel with the convection. The thermal resistance within the particle is important only if the conductivity of the solid is low, or if the diameter is large. Confining our attention to the opposite conditions for the present, the overall coefficient of heat transfer is stated mathematically in terms of the individual coefficients by the equation

$$h_m = h_r + \frac{h_{cp}}{1 + \gamma \dfrac{h_{cp}}{h_{cw}}}. \qquad (1)$$

This equation applies only to point conditions. Each of the individual coefficients and the area ratio, γ, varies with the position of the particle. In the case of particles falling through a cylindrical tube, γ may be expressed conveniently in terms of a dimensionless group which also enters into the radiation coefficient

$$\gamma = \frac{NA_p}{A_w} = \frac{\left(C \dfrac{\pi D^2}{4} dL\right)(4a)}{\pi D dL} = CDa.^* \qquad (2)$$

*The surface area of particles having convex surfaces is four times the projected area, cf. p. 32.

Little is known about the individual coefficients, as this problem in heat transfer has apparently received little attention. A discussion of the theory underlying the mechanism of heat transfer to falling particles will prove helpful in interpreting the experimental data and in extending the work to the practical design of large scale equipment.

5. *Heat Transfer to Small Particles by Convection.*—An exact analysis of this problem in convection is exceedingly difficult, if not impossible, because of the complicated hydrodynamics involved. Any solution of the problem must be made on the basis of simplifying assumptions which are valid at best only within certain limits. The resulting equations, therefore, must be recognized as approximations, and the nature of the assumptions should be clearly understood in order to avoid faulty application.

A. Simple Concept

One of the simplest concepts of the mechanism of heat transfer to falling particles is analogous to that recently proposed* for the absorption of gases by spray droplets. The assumptions, while admittedly crude, lead to an expression for the heat transfer coefficient of the same form as that given by the more rigorous attack on the problem to be described later. Furthermore, the numerical values calculated in this manner agree fairly well with those obtained from the experimental data, so that, as a first approximation, the equation has been recommended for design purposes.† The corresponding equation for mass transfer was likewise found to agree within ten percent with the observed values of the coefficient for the absorption of ammonia, sulphur dioxide, hydrogen sulphide, and carbon dioxide by falling droplets of various solutions.‡

In this concept, it is assumed that each particle makes contact with a tube of gas described by the trace of its periphery and of thickness s. This thickness represents the distance through which all of the heat above the temperature of the particle surface can be transmitted by conduction while the particular layer is in contact with the particle surface. As the velocity of the particle increases, s must decrease. In any case, s is small compared to the diameter of the particle, so that the volume of the tube is $\pi D_p L s$. In its simplicity, the tube of gas represents a zone in which the temperature is reduced to that of the surface of the particle, since complete temperature equilibrium is

*H. F. Johnstone and R. V. Kleinschmidt, Trans. Am. Inst. Chem. Engrs., vol. 34, p. 181, 1938.
†Univ. of Ill. Eng. Exp. Sta. Bul. 324, p. 79, 1940.
‡H. F. Johnstone and G. C. Williams, Ind. Eng. Chem., vol. 31, p. 993, 1939.

established within the allotted time. The layer obviously does not correspond to the so-called "laminar film," or "boundary layer" and is purely hypothetical.

The quantity of heat transferred across the tube from the ambient gas to the particle surface as the particle moves through a distance of one diameter is

$$\frac{\pi D_p^2 k}{s}(t_g - t_p)\frac{D_p}{V}.$$

From a heat balance for this segment of the tube,

$$\pi D_p^2 \frac{k}{s}(t_g - t_p)\frac{D_p}{V} = \pi D_p^2 s \rho_g c\,(t_g - t_p) \tag{3}$$

whence

$$s = \sqrt{\frac{kD_p}{V\rho_g c}}. \tag{4}$$

The heat transfer coefficient h_{cp} is

$$h_{cp} = \frac{k}{s} = \sqrt{\frac{kV\rho_g c}{D_p}}. \tag{5}$$

With the exception of the proportionality constant, here equal to unity, the term on the right is indeed identical with that obtained by Boussinesq in his classical treatment of thermal conduction to bodies submerged in moving fluids. A brief discussion of the fundamental equation and of the customary assumptions required for its solution will now be given, along with a solution which avoids the most doubtful approximation in the Boussinesq treatment.

b. Solution of the Fourier-Poisson Equation for a Spherical Particle

The Fourier-Poisson equation is a differential equation for thermal conduction in moving fluids. In rectangular coordinates its form is

$$c\rho\left(\frac{\partial t}{\partial \theta} + u\frac{\partial t}{\partial x} + v\frac{\partial t}{\partial y} + w\frac{\partial t}{\partial z}\right) = \frac{\partial}{\partial x}\left(k\frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial t}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial t}{\partial z}\right). \tag{6}$$

This equation itself is derived from the laws of hydrodynamics and thermodynamics. In its simple form, as written,* it applies strictly

*The assumptions underlying the Fourier-Poisson equation and its solution for various cases are discussed by T. B. Drew, Trans. Am. Inst. Chem. Engrs., vol. 26, p. 26, 1931.

only to an ideal incompressible fluid in which the forces of viscosity are negligible, and in which the kinetic energy changes are small compared to the total heat change. These restrictions themselves do not seriously limit the usefulness of the equation, as for many fluids, either gaseous or liquid, moving through a heat exchange system at relatively low velocities, the forces of viscosity are negligible and the pressure is nearly constant.

Application of Equation (6) to convection problems is made after determining, or assuming, the velocity distribution of the fluid, so that u, v, and w may be evaluated at all points in the system. The simplest case is that in which the velocity in all directions is zero. Such a condition would prevail approximately in heat transfer from a quiescent fluid to a very small particle. For the steady state in this case the solution* of Equation (6) requires that

$$q = 2\pi k D_p (t_s - t_\infty). \tag{7}$$

Thus, the heat transfer coefficient is

$$h_{cp} = \frac{2k}{D_p}. \tag{8}$$

This agrees with the recent observation made from experimental data by Meyer† that, for natural convection, the Nusselt number, $h_{cp}r/k$, approaches a constant value between 0.5 and 1.0 for small values of the Grashof group, $r^3\rho^2\beta g\Delta_0 t\ /\mu^2$. Meyer estimates that the heat transfer coefficient for a particle 0.0016 inch in diameter surrounded by air in a spray drier would be 240 B.t.u. (hr.) (sq. ft.) (deg. F.). With a temperature gradient of 200 deg. F., which customarily prevails in this apparatus, the rate of heat transfer would be 50 000 B.t.u. (hr.) (sq. ft. of particle surface). To obtain a rate of this magnitude by radiation alone would require a radiator temperature of about 2000 deg. F. This emphasizes the high rate of heat transfer attainable for small particles.

a. The Boussinesq Solution‡

In order to develop the Boussinesq solution of Equation (6) for spherical particles submerged in a moving fluid, it is desirable to consider first the case of the submerged flat plate. From this the

*The solution is best accomplished by writing the equation in spherical coordinates.
†P. Meyer, Trans. Inst. Chem. Engrs. London, vol. 15, p. 127, 1937.
‡J. Boussinesq, J. math. pures et appl., vol. 1, p. 285, 1905.

more complicated problem can be attacked by making additional assumptions.

Let the flat plate of width x_1 and of infinite length be located in the x, y plane with the leading edge at the y axis. The fluid moves in the x direction parallel to the surface. It is supposed that v and w are everywhere and at all times equal to zero, that u is constant and independent of z, and that k, c, and ρ are constants. These conditions limit the case to streamline flow over the plate, although the results should be valid as an approximation even when there may be turbulence near the trailing edge of the plate, but when the boundary layer as a whole is essentially laminar.

For the steady state, the foregoing conditions reduce the left side of Equation (6) to the single term, $c\rho u\partial t/\partial x$. Boussinesq simplified the equation further by assuming that the second partials with respect to x and v are negligible. For a long plate it is obvious that the flow of heat in the v direction must be negligible and, therefore, $\partial^2 t/\partial y^2$ will vanish. Neglect of the term $\partial^2 t/\partial x^2$, however, requires that the thermal conduction in the direction of flow also be negligible. This assumption will be shown to hold only within certain limits of the fluid flow defined by the Reynolds number.

The simplified equation now becomes

$$\frac{\partial t}{\partial x} = \frac{\alpha^2}{u} \cdot \frac{\partial^2 t}{\partial z^2} \tag{9}$$

where α^2 is the constant $k/c\rho$.

A solution of Equation (9) which satisfies the boundary conditions, $t = t_\infty$ for all positive values of x and for $z = \infty$, and $t = t_s$* for $z = 0$ for all values of x along the surface of the plate, is

$$\frac{t - t_\infty}{t_s - t_\infty} = 1 - \frac{2}{\sqrt{\pi}} \int_0^\xi e^{-\xi^2}\, d\xi \tag{10}$$

where

$$\xi = \frac{z}{2} \sqrt{\frac{u}{\alpha^2 x}}.$$

The rate at which heat is transferred from the fluid to the plate

*Although the equation may be solved for any variation of t with x, only the simple case of constant surface temperature need be considered here.

is found from the value of the temperature gradient at the surface. From Equation (10)

$$\left(\frac{\partial t}{\partial z}\right)_{z=0} = -(t_s - t_\infty)\sqrt{\frac{u}{\alpha^2 x\pi}}. \tag{11}$$

For the differential area $dxdy$ located at the distance x from the leading edge

$$dq = -k\left(\frac{\partial t}{\partial z}\right)_{z=0} dxdy = k(t_s - t_\infty)\sqrt{\frac{u}{\pi\alpha^2 x}}\, dxdy. \tag{12}$$

The overall rate of heat transfer to the strip of width dy and length x_1 is

$$q = k(t_s - t_\infty)\, dy \int_0^{x_1} \sqrt{\frac{u}{\alpha^2\pi x}}\, dx \tag{13}$$

$$= 2k(t_s - t_\infty)\, x_1 dy \sqrt{\frac{kc\rho u}{\pi x_1}}.$$

The average coefficient of heat transfer over the strip is

$$h_{av} = \frac{q}{(t_s - t_\infty)x_1 dy} = \frac{2}{\sqrt{\pi}}\sqrt{\frac{kc\rho u}{x_1}} = 1.13\sqrt{\frac{kc\rho u}{x_1}}. \tag{14}$$

Before proceeding further, it is well to examine the simplifying assumption made by Boussinesq. The assumption of negligible conduction in the direction of flow is valid only when the ratio of $\partial^2 t/\partial y^2$ to $\partial^2 t/\partial x^2$ is large.

If these derivatives are evaluated from Equation (10), the ratio of the first to the second is found to be

$$\left(\frac{2x}{z}\right)^2 \frac{1}{1 - \frac{3}{2}\left(\frac{2x}{z}\right)^2 (Re \cdot Pr)^{-1}\left(\frac{x_1}{x}\right)}.$$

Thus the approximation becomes more exact the larger the values of $Re \cdot Pr$ and of x/z. It is not necessary to consider small values of

x/z, however, because most of the heat transfer takes place near the plate. If only that region is included for consideration in which the temperature gradient $\partial t/\partial z$ is greater than one-tenth of its value at the surface, and if the smaller of the two derivatives may be neglected when its absolute value is one-tenth that of the larger, the approximation is valid for values of $Re \cdot Pr$ greater than eight. The principal assumption on which the Boussinesq solution is based is justified for diatomic gases when the Reynolds group exceeds 10.8, and for water (at ordinary temperatures) when the group exceeds 1.2.

Extension of the Boussinesq solution for Equation (6) to a cylinder submerged in a moving fluid is readily made by transformation from rectangular coordinates to the orthogonal curvilinear system represented by the stream function and the velocity potential of classical hydrodynamics. The only additional assumption required is that the motion of the fluid be irrotational, which in itself indicates that a velocity potential exists. By a suitable choice of the origin of coordinates, and after making the Boussinesq assumption of negligible conduction along a stream line, the form of the equation becomes identical with Equation (9). The boundary conditions are likewise the same and the solution is, therefore, similar to that already given for the flat plate.

For the three-dimensional case of a submerged body of revolution, the form of Equation (6) in the new coordinate system now described by stream lines ($\Psi =$ constant) and equipotential surfaces ($V\Pi =$ constant) is

$$\frac{\partial t}{\partial \Pi} = \frac{\alpha^2}{V}\left[\frac{\partial^2 t}{\partial \Pi^2} + \frac{2}{\left(\frac{\partial \Pi}{\partial x}\right)^2 + \left(\frac{\partial \Pi}{\partial y}\right)^2}\frac{\partial \Pi}{\partial x}\frac{\partial t}{\partial \Psi} + y^2\frac{\partial^2 t}{\partial \Psi^2}\right]. \qquad (15)$$

The x axis is the axis of revolution, and is parallel to the direction of flow, while y is the radial distance from the x axis.

In order to simplify Equation (15), the first and second terms in the bracket are assumed to vanish and the value of y is limited to its value at the surface of the body. Neglecting $\partial^2 t/\partial \Pi^2$ represents the usual assumption of negligible conduction along the stream lines, but the neglect of the second term and the limitation imposed on the value of y introduce new assumptions which must be examined. Solution of

the simplified equation as before gives the average heat transfer coefficient for the general case

$$h_{av} = M\sqrt{\frac{kV\rho c}{\lambda}} \tag{16}$$

where M is a constant dependent on the shape of the surface and λ is a characteristic linear dimension. For the sphere, the average coefficient is

$$h_{cp} = \frac{2}{\sqrt{\pi}}\sqrt{\frac{kV\rho c}{D_p}} = 1.13\sqrt{\frac{kV\rho c}{D_p}}. \tag{17}$$

We note that the coefficient is slightly higher than that calculated from the less rigorous Equation (5). As a matter of fact, the experimental data obtained in this study actually indicate that the proportionality constant should be less than one. Furthermore, Lévêque's* critical examination of the simplifying assumptions mentioned in the foregoing throws considerable doubt on the validity of the generalized Equation (16) for the transfer coefficient. It appears that the second term in the bracket of Equation (15) is actually negligible over most of the region surrounding the body in which the temperature gradient exists. On the other hand, the limitation on the value of y becomes an allowable approximation only when

$$y_{av} > 30\sqrt{\frac{(\Pi_1 - \Pi_0)\,\alpha^2}{V}}. \tag{18}$$

Here Π_1 and Π_0 are, respectively, the values of the velocity potential at the trailing and leading edge of the body. For potential flow around a sphere it can be shown† that

$$\Pi_1 - \Pi_0 = \frac{3D_p}{2}. \tag{19}$$

Also, for a sphere, y_{av} is approximately $D_p\,\pi$. Substitution of these

*A. Lévêque, Ann. mines, vol. 13, Ser. 2, p. 249, 1928.

†L. Prandtl and O. G. Tietjens, "Fundamentals of Hydro- and Aeromechanics," p. 151, McGraw-Hill Book Co., New York, 1934.

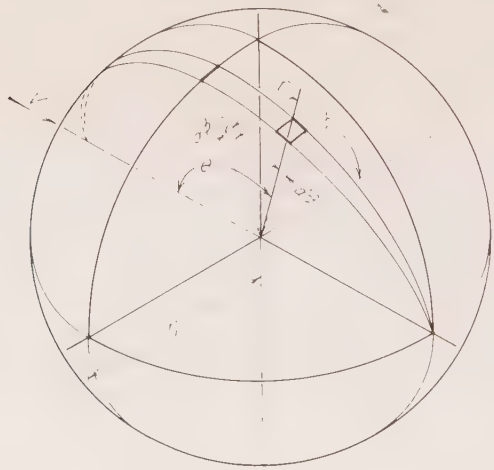

FIG. 1. GEOMETRICAL REPRESENTATION OF PROBLEM OF HEAT TRANSFER TO A SPHERE

values in the inequality shows that the Boussinesq assumptions are allowable when

$$\frac{D_p V}{\alpha^2} = \left(\frac{D_p V \rho}{\mu}\right)\left(\frac{c\mu}{k}\right) > 13\ 400. \tag{20}$$

In such ranges of flow there is little doubt that the entire analysis will break down due to the departure from ideal flow conditions.

b. Solution in Spherical Coordinates in Terms of Bessel Functions

The Boussinesq solution of the Fourier-Poisson equation for heat transfer to a sphere assumed that a velocity potential exists for the flow around the body. By making a slightly different assumption in regard to the flow pattern it is possible to solve the equation directly in terms of Bessel functions which can be evaluated.

For the steady state, Equation (6) may be written in the spherical coordinates, r, θ, and φ (see Fig. 1) as follows:

$$v_r \frac{\partial t}{\partial r} + \frac{v_\theta}{r}\frac{\partial t}{\partial \theta} + \frac{v_\varphi}{r \sin\theta}\frac{\partial t}{\partial \varphi} =$$

$$\alpha^2\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial t}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 t}{\partial \varphi^2}\right] \tag{21}$$

v_r, v_θ, and v_φ are components of velocity in the direction of increasing r, θ, and φ, respectively.

We shall assume that the velocity of the fluid around the sphere is everywhere parallel to the surface and equal to V. As a result of symmetry about the axis of revolution, v_r, v_φ, and $\partial^2 t/\partial\varphi^2$ are zero, and $v_\theta = V$. The second term in the bracket represents the accumulation of heat in an element of fluid due to heat transfer in the direction of increasing θ. Since the streamlines have been assumed to be parallel to the surface, this represents the net conduction along a streamline, and should be negligible according to the usual Boussinesq assumption. The equation then becomes

$$\frac{\partial t}{\partial \theta} = \frac{\alpha^2}{Vr}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right). \tag{22}$$

A solution of Equation (22) may be assumed to have the form

$$\frac{t - t_s}{t_\infty - t_s} = F(\theta)\cdot G(r) \tag{23}$$

where $F(\theta)$ is a function of θ alone and $G(r)$ is a function of r alone. Differentiation of Equation (23) and substitution in Equation (22) gives

$$\frac{1}{F}\frac{dF}{d\theta} = \frac{\alpha^2}{VG}\left[2\frac{dG}{dr} + r\frac{d^2G}{dr^2}\right]. \tag{24}$$

Since each side of this equation is a function of only one independent variable, each may be set equal to a constant, say $-b^2$. Then the forms of the functions $F(\theta)$ and $G(r)$ are found by solving the ordinary differential equations:

$$\frac{dF}{d\theta} + b^2F = 0 \tag{25}$$

and

$$r\frac{d^2G}{dr^2} + 2\frac{dG}{dr} + \frac{b^2V}{\alpha^2}G = 0. \tag{26}$$

The first of these has the solution

$$F(\theta) = Be^{-b^2\theta} \tag{27}$$

and the second,

$$G(r) = \frac{\alpha}{b\sqrt{rV}} J_1\left(\frac{2b\sqrt{rV}}{\alpha}\right) \tag{28}$$

where J_1 is the Bessel function of the first kind and of the first order.

Since the original equation is linear in G and its derivatives, the sum of any number of particular solutions is also a solution, or

$$\frac{t - t_s}{t_\infty - t_s} = \sum_{i=0}^{\infty} B_i e^{-b_i^2\theta} \frac{\alpha}{b_i\sqrt{rV}} J_1\left(\frac{2b_i\sqrt{rV}}{\alpha}\right). \tag{29}$$

The arbitrary constants B_i and b_i may be determined from the boundary conditions which require, first, that $t = t_s$ for $\theta > 0$ and $r = r_1$ and, second, that $t = t_\infty$ for $\theta = 0$ and $r > r_1$. From the first condition,

$$0 = J_1\left(\frac{2b_i\sqrt{rV}}{\alpha}\right). \tag{30}$$

Thus, the successive values of the constant become

$$b_i^2 = 0, \quad \frac{\eta_1^2}{2Re \cdot Pr}, \quad \frac{\eta_2^2}{2Re \cdot Pr}, \text{ etc.}$$

The values of η, which are the roots of $J_1(\zeta) = 0$, are given in standard tables.*

From the second boundary condition

$$1 = \sum_{i=0}^{\infty} B_i \frac{\alpha}{b_i\sqrt{rV}} J_1\left(\frac{2b_i\sqrt{rV}}{\alpha}\right). \tag{31}$$

The values of B_i may now be determined by comparing the coefficients with those of similar terms of a Fourier-Bessel series† for the expansion of $\sqrt{r}$:

$$\sqrt{r} = \sum_{i=0}^{\infty} \frac{\alpha}{b_i\sqrt{V}\, J_2\left(\frac{2b_i\sqrt{rV}}{\alpha}\right)} J_1\left(\frac{2b_i\sqrt{rV}}{\alpha}\right). \tag{32}$$

*E. Jahnke and F. Emde, "Funktionentafeln," Teubner, Berlin, 1909.

†F. Bowman "Introduction to Bessel Functions," p. 108-11, Longmans, Green and Co., London, 1938.

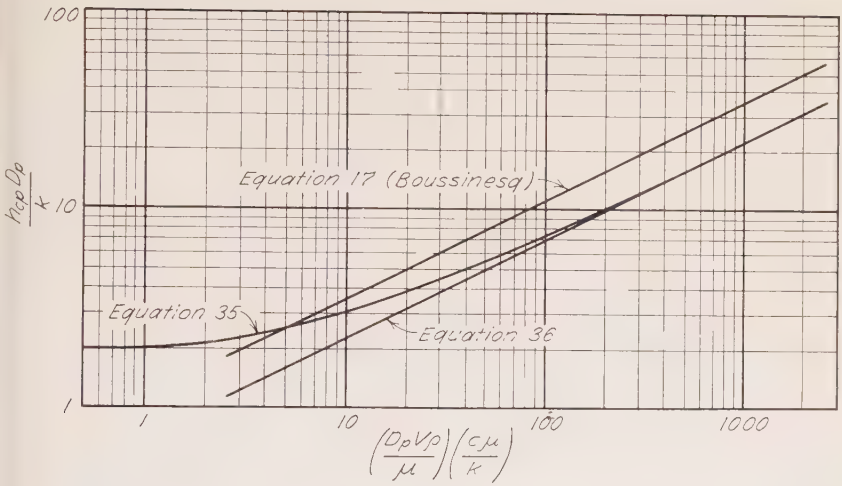

FIG. 2. COMPARISON OF EQUATIONS OF HEAT TRANSFER TO A SPHERICAL PARTICLE

The final solution of Equation (22) then becomes

$$\frac{t - t_s}{t_\infty - t_s} = \sum_{i=0}^{\infty} \frac{\alpha}{b_i J_2\left(\frac{2b_i\sqrt{rV}}{\alpha}\right)} \frac{e^{-b_i^2\theta}}{\sqrt{rV}} J_1\left(\frac{2b_i\sqrt{rV}}{\alpha}\right). \quad (33)$$

The total rate of heat transfer across the surface of the hemisphere is

$$q = -\int_0^{\pi}\int_0^{\pi} k\left(\frac{\partial t}{\partial r}\right)_{r=r_1} r^2 \sin\theta \, d\varphi d\theta$$

$$= k\pi r_1 (t_\infty - t_s) \sum_{i=0}^{\infty} \frac{1 + e^{-\pi b_i^2}}{1 + b_i^4}. \quad (34)$$

The average heat transfer coefficient then is

$$h_{cp} = \frac{k}{D_p} \sum_{i=0}^{\infty} \frac{1 + e^{-\pi b_i^2}}{1 + b_i^4}. \quad (35)$$

The series converges fairly rapidly. For $Re \cdot Pr = 10$ the sixth term is only 0.1 per cent of the sum, while for $Re \cdot Pr = 100$, seventeen terms are required for this percentage.

Equation (35) is plotted in Fig. 2 along with Equations (17) and (36). It is interesting to note that, according to Equation (35), the Nusselt group, $h_{cp}D_p/k$, approaches the value of 2.0 as the Reynolds number approaches zero, as required by Equation (8) for the heat conduction to very small spheres in a quiescent fluid. At high Reynolds numbers the curve approaches the value given by

$$h_{cp} = 0.714\sqrt{\frac{kV\rho c}{D_p}} \quad . \tag{36}$$

This is a limiting value which may be derived by integrating Equation (13) for heat transfer to a submerged flat plate over a spherical surface. Such an integration is justifiable when the boundary layer is very thin. In such a case the surface of the sphere may be conceived of as being made up of many narrow flat strips each of angular width $d\phi$, and extending from the leading point to the trailing point. The diagrammatic representation of this problem is also shown in Fig. 1. We again assume that the velocity of the fluid elements is everywhere constant and parallel to the surface of the sphere. From Equation (14) the rate of heat transfer to an element of area, $r_1^2 \sin\theta d\theta d\varphi$, located at a distance, $r_1\theta$ from the leading point of the sphere is

$$dq = (t_s - t_\infty)\sqrt{\frac{c\rho Vk}{\pi}}\,\frac{r_1 \sin\theta\, d\theta d\varphi}{\sqrt{r_1\theta}}. \tag{38}$$

Integration with respect to θ may be performed after expansion of $\sin\theta$ in a series. This gives

$$q = (t_s - t_\infty)\frac{1.790}{\sqrt{2\pi}}\sqrt{\frac{c\rho Vk}{D_p}} \cdot 2\pi r^2 \tag{39}$$

and the average heat transfer coefficient becomes that given by Equation (36).

Not only do the values of h_{cp} from Equation (35) agree more nearly with the experimental results than the Boussinesq Equation (17), but also there is reason to believe that the assumed pattern of flow around the sphere corresponds more nearly to actual conditions in the region near the surface in which the resistance to heat transfer

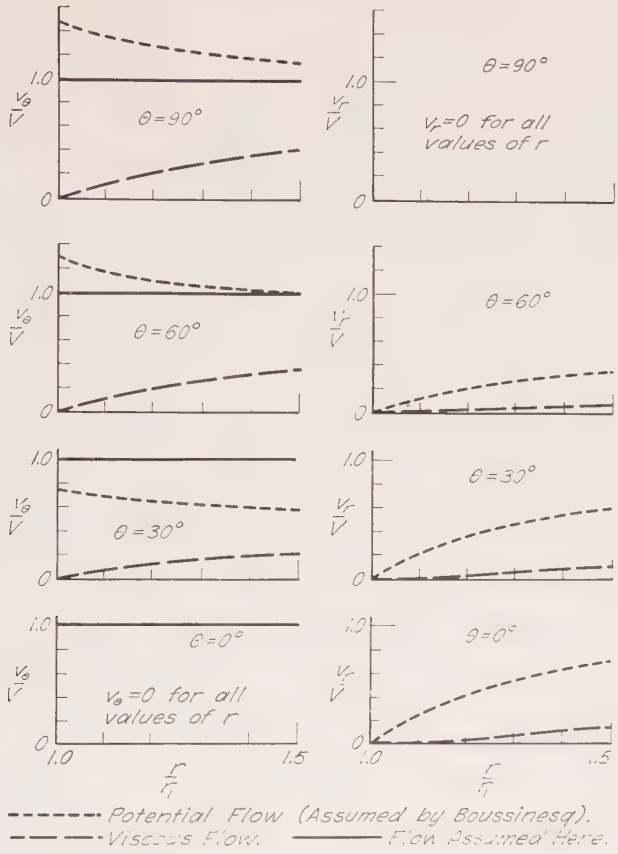

FIG. 3. ASSUMED COMPONENTS OF FLUID VELOCITY IN NEIGHBORHOOD OF A SPHERE

exists* than that required by the theory of potential flow. The components of velocity are compared with those calculated for potential flow and for viscous flow around a sphere in Fig. 3. The latter are derived from the Stokes' equations† which neglect entirely the forces of inertia. At a point on the surface 90 degrees from the leading point, according to the potential flow theory, the fluid has a tangential velocity of $1.5V$, while, according to the theory of viscous flow, the

*The effective thickness of a uniform fluid layer surrounding a sphere in which all of the resistance to heat transfer may be considered to exist, expressed as a fraction of the radius of the sphere, is approximately $2k/h_{cp}\ D_p$. According to Equation (35) this group is never greater than one and at a value of $Re \cdot Pr$ equal to 100 the value is 0.3. Therefore the distance in question cannot be greater than the radius of the sphere.

†Leigh Page, "Introduction to Theoretical Physics," p. 233. D. van Nostrand, Inc., New York. 1928.

velocity must be zero. Actually the velocity is zero at the surface and increases rapidly within a narrow region to a value nearly equal to V. The radial component is zero according to either theory. Thus, at this location, the velocity has approximately the value assumed for the foregoing solution. At other points on the surface the assumption of the constant tangential velocity is not far in error.

6. *Radiation to Clouds of Particles.*—Heat transfer by radiation from a solid surface to small opaque particles is similar in principle to radiation to an absorbing gas. When a radiant beam enters a cloud of particles, those particles near the front of the cloud have a greater chance of intercepting the beam than have those in the interior or at the rear of the cloud. This is equivalent to saying that the average intensity of the radiation varies through the cloud. It is necessary to know the variation of intensity with distance along the beam in order to calculate the transfer of radiant energy to particle clouds of various shapes. The problem is analogous to that arising in gas radiation. In fact, the equations used for the calculation, with the introduction of new constants, will be similar to those used by Nusselt* for calculating radiation to different gas shapes.

For particles which absorb the incident radiation completely the problem of variation of intensity with distance is that of finding at any point the average fraction of the beam blanked out by particles along its path. This is equivalent to finding the covering power of a number of small discs tossed at random on a plate. The analysis has been made by Haslam and Hottel† as follows:

When n discs, each of area a, have been tossed on a plate of area A, the fraction covered will be f_n. The probability that the $(n+1)$st disc will fall on uncovered area is $1 - f_n$. On the average, its effective covering power will be a $(1 - f_n)$. For $n+1$ discs the fraction covered is

$$f_{n+1} = f_n + (1 - f_n)\left(\frac{a}{A}\right) = f_n\left(1 - \frac{a}{A}\right) + \frac{a}{A}. \qquad (39)$$

In a similar manner, the fraction covered by $(n+2)$ discs is

$$f_{n+2} = f_{n+1}\left(1 + \frac{a}{A}\right) + \frac{a}{A} = f_n\left(1 - \frac{a}{A}\right)^2 + 2\left(\frac{a}{A}\right) - \left(\frac{a}{A}\right)^2$$

*W. Nusselt, Forschungsarbeiten Geb. Ing., No. 264, p. 40, 1923; Zeit. Ver. Deut. Ing., vol. 70, p. 763, 1926.

†R. T. Haslam and H. C. Hottel, Trans. Am. Soc. Mech. Eng. (FSP 50-3), vol. 50, p. 9, 1928.

or

$$f_{n+2}-1=f_n\left(1-\frac{a}{A}\right)^2-\left[1-\frac{2a}{A}+\left(\frac{a}{A}\right)^2\right]=(f_n-1)\left(1-\frac{a}{A}\right)^2. \quad (40)$$

By mathematical induction, for $(m + n)$ discs

$$f_{m+n}-1=(f_n-1)\left(1-\frac{a}{A}\right)^m. \quad (41)$$

When $n = 0$,

$$f_m=1-\left(1-\frac{a}{A}\right)^m. \quad (42)$$

If C is the number of solid particles per unit volume in the space above A and l is the thickness of the cloud above A, the number of particles, m, projected on A is ClA. Therefore

$$f_m=1-\left(1-\frac{a}{A}\right)^{ClA}. \quad (43)$$

When the area of each particle is very small compared to the area of the radiating surface,

$$\begin{aligned} f_m &= \lim_{a/A\to 0}\left[1-\left(1-\frac{a}{A}\right)^{ClA}\right] \\ &= 1-\left[\lim_{a/A\to 0}\left(1-\frac{a}{A}\right)^{A/a}\right]^{Cla} \\ &= 1-e^{Cla}. \end{aligned} \quad (44)$$

From Equation (44) the relationship between the average intensity at point l and the intensity at $l = 0$ is

$$I_l=I_0e^{-Cla}. \quad (45)$$

For particles which do not absorb completely, but which have a high emissivity so that the second order radiation is negligible, Equation (45) becomes

$$I_l=I_0e^{-\epsilon_p Cla}. \quad (46)$$

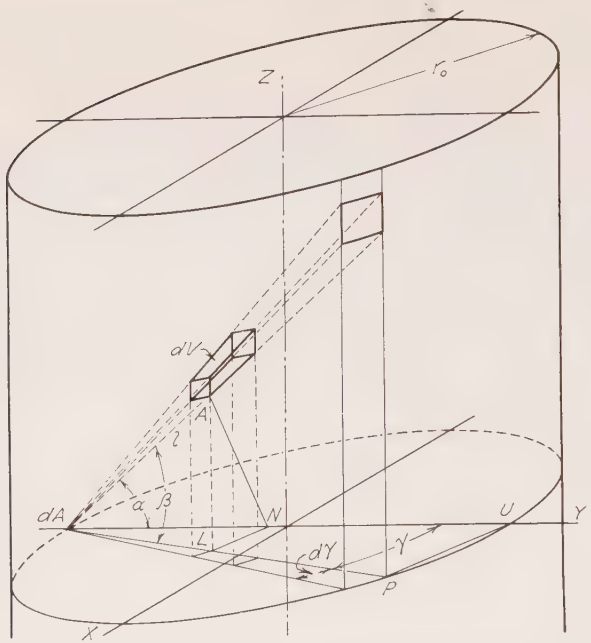

FIG. 4. GEOMETRICAL ANALYSIS OF RADIATION FROM WALLS OF AN INFINITE CYLINDER TO CLOUD OF PARTICLES

This equation is similar to Beer's law for the absorption of light by liquids and gases except that here the proportionality factor may be calculated from the size of the particle, while in Beer's law it must be evaluated experimentally.

Equation (46) may be used for calculating the absorptivity of a cloud of particles of a given shape and concentration. Referring to Fig. 4, the rate at which heat radiating from a surface element dA, strikes an element of volume dV, located in a cloud of particles at a distance l from dA and subtending the solid angle $d\omega$ at dA is given by

$$dq_1 = I_n \cos \alpha \, e^{-\epsilon_p C l a} dA \, d\omega. \tag{47}$$

The average intensity of the ray at the front face of the volume element is obtained by dividing Equation (47) by the cross sectional area ($l^2 d\omega$) of the ray at the point l.

$$I_l = I_n \frac{\cos \alpha \, e^{-\epsilon_p C l a}}{l^2} dA. \tag{48}$$

Now consider that the element of volume, dV, is subdivided into cylindrical elementary rays, each having a length dl and a cross sectional area dA'. The quantity of radiation entering each of these elementary cylinders has the value $I_l dA'$. From Equation (46) the fraction $\epsilon_p Cadl$ of this amount is absorbed on passage through the cylinder. The total amount dq_2 absorbed by the volume element dV is $\epsilon_p CaI_l\ dV$, or

$$dq_2 = \frac{\epsilon_p CaI_n \cos\alpha \cdot e^{-\epsilon_p Cla} dA dV}{l^2}. \tag{49}$$

The total radiation from a solid wall which is absorbed by a cloud of particles is obtained by integration of Equation (49) over the area of the radiation wall and the volume of the cloud,

$$q_2 = \epsilon_p CaI_n \int_{\text{vol.}} \int_{\text{area}} \frac{\cos\alpha \cdot e^{-\epsilon_p Cla} dA dV}{l^2}. \tag{50}$$

The total rate at which energy is emitted in all directions from the surface is $I_n \pi A$. The absorptivity of the cloud of particles is the fraction of this total emission which is absorbed, that is,

$$\epsilon_c = \frac{q_2}{I_n \pi A} = \frac{\epsilon_p Ca}{\pi A} \int_{\text{vol.}} \int_{\text{area}} \frac{\cos\alpha \cdot e^{-\epsilon_p Cla}}{l^2} dA dV. \tag{51}$$

The integration of Equation (51) has been performed by Nusselt* for the case of the sphere and the case of the infinite cylinder. For the former,

$$\epsilon_c = 1 - \frac{1}{2(\epsilon_p Cr_0 a)^2} + \frac{1 + 2\epsilon_p Cr_0 a}{2(\epsilon_p Cr_0 a)^2} e^{-2\epsilon_p Cr_0 a}. \tag{52}$$

For the infinite cylinder

$$\epsilon_c = 1 - \frac{4}{\pi} \int_0^{\pi/2} \int_0^{\pi/2} e^{\frac{-2\epsilon_p Cr_0 a \cos\gamma}{\cos\beta}} \cos^2\beta\ \cos\gamma\ d\beta d\gamma. \tag{53}$$

The integral in Equation (53) was evaluated graphically.

*W. Nusselt, Zeit. Ver. Deut. Ing. vol. 70, p. 763, 1926.

TABLE 1

ABSORPTIVITY OF CLOUDS OF PARTICLES IN FURNACES OF VARIOUS SHAPES

$2\epsilon_p Cr_0 a$	Absorptivity ϵ_c			$2\epsilon_p Cr_0 a$	Absorptivity ϵ_c		
	Sphere	Infinite Cylinder	Between Infinite Parallel Plates*		Sphere	Infinite Cylinder	Between Infinite Parallel Plates*
0	0	0	0	3.50	0.8547	0.9293	0.9901
0.10	0.0642	0.0932	0.1674	4.00	0.8865	0.9460	0.9945
0.20	0.1243	0.1767	0.2961	4.50	0.9073	0.9580	0.9969
0.25	0.1520	0.2149	0.3506	5.00	0.9232	0.9665	0.9983
0.30	0.1792	0.2512	0.3999	5.50	0.9356	0.9728	0.9989
0.40	0.2308	0.3170	0.4854	6.00	0.9454	0.9776	0.9994
0.50	0.2748	0.3764	0.5568	6.50	0.9532	0.9814	0.9996
0.75	0.3835	0.5000	0.6905	7.00	0.9595	0.9843	0.9998
1.00	0.4715	0.5957	0.7806	7.50	0.9646	0.9868	0.9999
1.25	0.5451	0.6709	0.8428	8.00	0.9688	0.9885	1.0000
1.50	0.6066	0.7298	0.8866	8.50	0.9724	0.9901	1.0000
1.75	0.6596	0.7769	0.9175	9.00	0.9753	0.9913	1.0000
2.00	0.7030	0.8142	0.9397	9.50	0.9778	0.9927	1.0000
2.50	0.7719	0.8689	0.9674	10.00	0.9800	0.9940	1.0000
3.00	0.8220	0.9048	0.9822	∞	1.0000	1.0000	1.0000

*Here r_0 represents the half distance between the plates.

For the case of infinite parallel plates, Jakob* obtained the equation

$$\epsilon_c = 1 - 2\int_0^{\pi/2} e^{\frac{-2\epsilon_p Cr_0 a}{\cos \alpha}} \sin \alpha \cdot \cos \alpha \cdot d\alpha. \qquad (54)$$

Here r_0 represents one-half of the distance between the plates. The integral in Equation (55) may be evaluated in terms of the exponential integral

$$Ei(-p) = \int_0^p \frac{e^{-x}}{x}\, dx$$

for which values are listed in standard tables.† The equation then becomes

$$\epsilon_c = 1 - (1 - 2\epsilon_p Cr_0 a)\, e^{-2\epsilon_p Cr_0 a} + (2\epsilon_p Cr_0 a)^2 Ei\,(-2\epsilon_p Cr_0 a). \qquad (55)$$

All of these shapes are of practical importance in furnace design. The values of ϵ_c for the three cases are listed in Table 1 and are plotted in Fig. 5. Inspection of the curves shows that for the same value of

*M. Jakob, "Der Chemie-Ingenieur," vol. 1, No. 1, pp. 299-303, Akad. Verlag., Leipzig, 1933.

†Jahnke-Emde, "Functionentafeln," *loc. cit.*

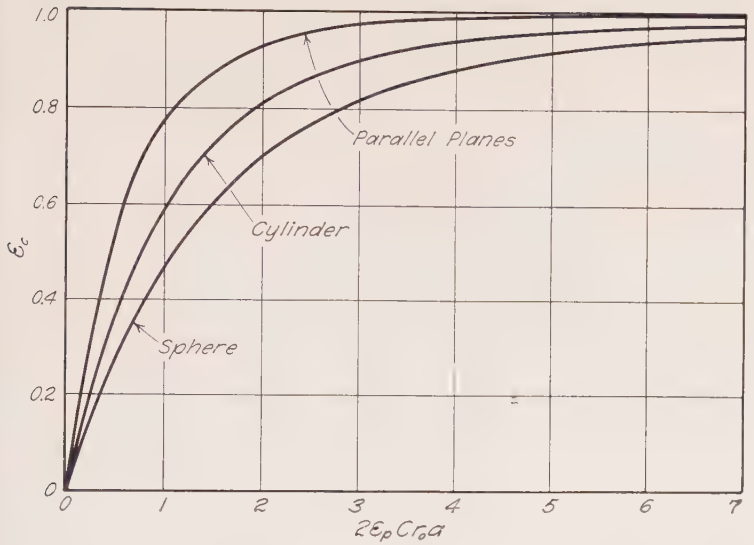

FIG. 5. ABSORPTIVITY OF CLOUDS OF PARTICLES IN FURNACES OF VARIOUS SHAPES

$2\epsilon_p C r_0 a$, or $\epsilon_p C D a$, a cloud of particles enclosed between infinite parallel planes has the greatest absorptivity. This may be explained qualitatively by the fact that the average length of a radiant beam is longest for the case of infinite parallel planes because of infinite dimensions in two directions. The absorptivity of the cloud enclosed in an infinite cylinder is higher than that for a sphere because the cylinder has infinite dimensions in one direction. The absorptivity for other shapes of clouds encountered in practice may be estimated from Fig. 5 by interpolating between the curves shown. For example, the value for a cube should fall between the curve for an infinite cylinder and that for a sphere, since the average length of a radiant beam is less than that in the cylinder but greater than that in a sphere. The absorptivity for a long shaft of rectangular cross section should fall between the two upper curves.*

*In a recent paper, Hottel and Egbert show that the effect of the gas shape on radiation of furnace gases may be approximated by using an effective average beam length. On this basis, the absorptivity of a hemisphere of a cloud of particles receiving radiation from a spot on the center of the base of the hemisphere of radius r is $1 - e^{-\epsilon_p C r a}$. When the gas shape is other than a hemisphere, the same formulation suffices provided the radius of the hemisphere is replaced by the effective beam length, l, which, for small values of the quantity $\epsilon_p C r a$, is four times the mean hydraulic radius, *i.e.*, four times the gas volume divided by the area of the bounding walls. This is exact as ϵ_c approaches zero. Comparison with the theoretical curves of Figure 5 shows that the suggested approximation is good up to values of ϵ_c of 0.2 but above this value considerable deviation occurs. For high values of the group analogous to $\epsilon_p C r a$ in gas radiation, Hottel and Egbert recommend using 85% of the effective average beam length obtained as above. This appears to be satisfactory for ϵ_c greater than 0.6. While this method of approximation has the advantage of conforming to established engineering practice in the calculation of heat transfer from radiating gas masses, it lacks the theoretical background of Equations (52), (53) and (54). (H. C. Hottel and R. B. Egbert, Preprint of paper presented before Am. Soc. Mech. Eng., New York, Dec. 1940.)

The area-emissivity factor F_{AE} is given by

$$F_{AE} = \frac{1}{\frac{1}{\epsilon_w} + \frac{1}{\epsilon_c} - 1} \tag{56}$$

where ϵ_w is the absorptivity of the wall. Since ϵ_c is generally small compared to ϵ_w, F_{AE} may be taken as equal to ϵ_c. The coefficient of radiant heat transfer to *clouds of particles* expressed on the basis of the area of the particles then is given by

$$h_r = \frac{0.172\epsilon_c \left[(T_w/100)^4 - (T_p/100)^4\right]}{\gamma\,(T_w - T_p)}. \tag{57}$$

7. *Heat Transfer from Wall to Gas.*—When a cloud of particles falls through a heated tube, three factors produce convection within the tube, viz., the normal convective forces due to density differences along the wall, the increased convection due to the introduction of cold particles at the top of the tube, and the disturbance due to the passage of the particles through the gas. Consequently, the rate of heat transfer from the wall to the gas is somewhat greater than that due to natural convection. Because of the complexity of the problem no attempt has been made to reduce it to a mathematical basis.

In the summary of the work previously published,* the Rice equation† for the heat transfer coefficient for natural convection from a long vertical pipe was recommended as a rough approximation. In this the coefficient h_{cw} is given as a function of the Grashof group.

$$h_{cw} = 0.115\,\frac{k}{D}\left[\frac{D^3\rho_g^2 g\beta\,(t_w - t_g)}{\mu^2}\right]^{1/3}. \tag{58}$$

It was realized that the values of h_{cw} from Equation (58) would be quite conservative. From the treatment of the experimental data which follows it appears that the actual values are from two and one-half to four times those calculated from the Rice equation. The coefficient is apparently independent of the wall temperature and of the particle size but increases with the diameter of the furnace.

*Univ. of Ill. Eng. Exp. Sta. Bul. 324, p. 74, 1940.
†W. H. McAdams, "Heat Transmission," p. 254, McGraw-Hill Book Co., New York, 1933.

TABLE 2
PRINCIPAL DIMENSIONS OF FURNACES USED IN EXPERIMENTAL WORK

	Small Furnace	Large Furnace
Inside diameter of tube, in.	1.61	3.76
Outside diameter of tube, in.	1.90	4.00
Inside diameter of refractory lining, in.	2.5	5.0
Total length of tube, in.	54.5	120.0
Heated length of tube, in.	48.0	110.0
Distance from top of calorimeter to bottom of heated section, in.	2.0	4.0
Distance from feeder to top of heated section, in.	4.0	17.0
Location of thermocouples above bottom of tube, in.	8, 20, 32, and 44	27.6, 49.2, 70.8, 92.4, and 114.0

III. EXPERIMENTAL WORK

8. *Introduction.*—The object of the experimental work was to provide accurate data covering a sufficient number of conditions so that the individual heat transfer coefficients could be determined. The agreement of these with the theoretical equations or with empirical correlations should justify extrapolation to large scale design. To this end, measurements were made on the rate of heat transfer to particles of known average sizes of several materials falling through heated tubes at known wall temperatures. The rate at which the particles were introduced was varied over a considerable range as this proved to be the important variable by which the data could be analyzed to calculate the individual coefficients. The effect of changing from air to carbon dioxide as the ambient gas was also studied.

The general method of the study consisted in adjusting the walls of a tubular furnace to constant temperature and then introducing the fractionally screened particles at a constant rate. The quantity of heat transferred and the temperature of the particles were determined by collecting the material in a calorimeter immediately below the furnace and observing the temperature rise of the water therein. The particles studied were sand, carborundum, and aloxite. These materials were selected because they are inert to air at high temperatures and do not react with water. Furthermore, their heat conductivities and absorptivities differ considerably.

9. *Heat Transfer Measurements.*—Two sizes of furnaces were used. The essential dimensions of these are given in Table 2. Details

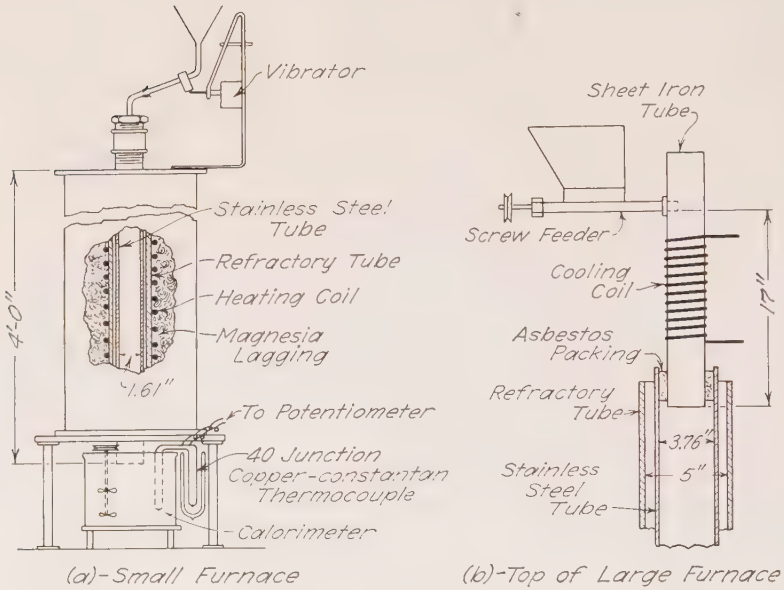

Fig. 6. Apparatus Used for Heat Transfer Measurements

of the small furnace and the top of the large furnace are shown in Fig. 6. In both cases stainless steel tubes were used and thermocouples were peened into the walls from the outside. The smaller furnace was heated electrically by resistance coils wound in four sections on a corrugated alundum sleeve. Uniformity of temperature was maintained by adjusting the external resistance of the heating elements. A typical temperature distribution along the inside of the tube is shown in Table 3.

The large furnace was heated by natural gas. The stainless steel tube was surrounded by cylindrical fire-brick tile, 1 in. thick, for protection against the direct action of the flames. The outer wall of the furnace was constructed of 4½-in. fire-brick with nine burner ports arranged for tangential firing. Control of this furnace was manual. The temperature of the tube wall indicated by the six thermocouples was recorded by a multipoint recording pyrometer. Variation along the tube did not exceed 25 deg. F. during a run.

The feed to the small furnace was introduced into the center through a glass tube. The rate was controlled by an electric vibrating feeder. The feed to the large furnace was introduced from a hopper by means of a motor-driven screw. This discharged into a water-

Table 3
Typical Temperature Distribution in Small Furnace

Distance Below Top of Heating Zone, in.	Temperature deg. F.	Distance Below Top of Heating Zone, in.	Temperature deg. F.
2	707	26	1047
5	992	29	1052
8	1105	32	1072
11	1130	35	1092
14	1085	38	1112
17	1075	41	1107
20	1075	44	1030
23	1067	47	812

Average temperature, omitting top and bottom points = 1075 deg. F.
Average temperature from four wall thermocouples = 1080 deg. F.

cooled section of vertical pipe fitted in the center of the stainless steel tube through asbestos packing. This arrangement was necessary in order to avoid heating the particles in the screw feed by conduction.

The calorimeter used was a four-liter Dewar flask fitted with a wooden top through which a stirrer was attached. With both furnaces, the tube extended about one inch through a hole in the top of the calorimeter. A forty-junction copper-constantan thermocouple, with one set of junctions in ice, was used to measure the temperature rise. The thermocouple was carefully calibrated against a Beckmann thermometer. A Leeds and Northrup students' type potentiometer with extended lower scale was used to measure the thermocouple e.m.f. The water equivalent of the calorimeter, thermocouple, and stirrer was determined electrically several times during the work.

The general procedure adopted for the runs was as follows: A known small quantity of the powder was introduced into the hopper of the feeder. When the temperature of the furnace was uniform and constant and the rate of temperature rise of the calorimeter was constant, the feed was started. The rate was maintained constant until the hopper had discharged to a fixed mark. In order to avoid a draft action through the tube, the hopper was not allowed to empty completely. The elapsed time was measured with a stop watch. The net increase in temperature of the calorimeter was noted after the rise had again reached a constant rate. The wall temperature was taken as the average of that observed just preceding and just after the run. In general, these did not differ by more than 5 deg. F.

Measurements were made in this way in the small furnace at wall temperatures from 575 to 1050 deg. F., and at feed rates from 3 to 50 lbs. per min. per sq. ft. Three particle sizes of sand and two of carborundum and aloxite were used. Two series of runs were made

with sand at wall temperatures of 725 and 1050 deg. when the air in the furnace had been replaced by carbon dioxide. In the large furnace two series of runs were made at a wall temperature of 1000 deg. F. using two screen fractions of sand.

10. *Secondary Measurements.*—In order to calculate the overall heat transfer coefficients from the measurements it was necessary to have accurate data on the average diameter, the projected area, and the rates of fall of the particles. This necessitated a secondary study in which these quantities were measured accurately by improved methods.

A. Average Particle Size

The arithmetic average "diameter" of the particles in each screen fraction was determined by observation under a petrographic microscope equipped with a movable stage. The "true average diameter" was obtained by securing random orientation on the microscope slide by using a method suggested by Tooley.* A thin layer of a gum arabic solution in glycerine and water was spread uniformly over the slide. Pin scratches in the layer were observed under the microscope until the film reached a stiff jelly-like consistency. The material to be measured was mixed thoroughly and a small quantity was taken from the whole with a spatula. This was spread on a paper which was then held a short distance above the slide and tapped lightly to transfer the particles with a uniform distribution. The orientation of the particles remained fixed as they fell on the gum.

Several hundred particles on each slide were traversed under the microscope from several directions. The average diameter was found by dividing the reading on the micrometer screw by the number of particles measured in each traverse. The average for all the traverses was then taken as the "true average diameter," D_p.

B. Average Projected Area

Tooley* has shown that for particles having plane or convex surfaces the surface area available for heat transfer is four times the average projected area obtained for all possible orientations of the particles. The slides prepared as described were placed on a projecting microscope and the images on a ground glass plate were traced on paper. The projected areas of individual particles were then measured with a planimeter. The areas of 50 to 100 particles from each sample

*F. V. Tooley, Ph.D. Thesis in Engineering, University of Illinois, 1939.

TABLE 4
SUMMARY OF MEASUREMENTS OF DIAMETER, MASS, AND DENSITY OF PARTICLES USED IN HEAT TRANSFER WORK

Screen Size	Average Diameter D_p mm.	Average Particle Mass w_p gm. $\times 10^4$	Density gm. per cc.	Volume Constant K	Average Projected Area a sq. in. $\times 10^{-2}$	
					Experimental	Calculated $\pi D_p^2/4$
A. Sand						
30–40	0.545	1.865	2.65	0.435	2.50	2.15
40–50	0.436	0.941	....	0.429	1.40	1.39
60–80	0.326	0.35	....	0.431	0.904	0.904
				Av. 0.432		
B. Carborundum						
No. 40	0.502	1.214	3.21	0.290	1.92	1.98
No. 60	0.344	0.386	....	0.351	1.02	0.935
				Av. 0.320		
C. Aloxite						
No. 46	0.485	1.362	3.92	0.304	1.88	1.85
No. 60	0.344	0.581	....	0.344	0.949	0.935
				Av. 0.324		

were averaged. This method of determining the surface area is obviously not applicable to particles having concave surfaces. With none of the materials studied was this condition serious.

C. Average Mass and Volume of Particles

The average mass of a particle was found by counting and weighing from one thousand to three thousand particles out of each fraction.

The average volume of the particles was calculated from the average mass and the density. The latter was found by displacement of water in a volumetric flask.

From the average volume and diameter of the particle the "volume-shape factor" K was calculated from

$$K = \frac{v_p}{D_p^3}. \tag{59}$$

Values of the factor were approximately constant for different sizes of the three materials studied but were much larger for the sand than

for the carborundum or aloxite particles. In all cases the coefficient was lower than $\pi/6$ which would be the value for spheres.

A summary of the data obtained in these measurements is given in Table 4.

TABLE 5

SUMMARY OF RESULTS ON DRAG COEFFICIENTS FOR SAND, CARBORUNDUM, AND ALOXITE PARTICLES

Screen Size	Average Diameter of Particles D_p mm.	Observed Terminal Velocity V_t ft. per sec.	Drag Coefficient C_D	Reynolds Number* $D_p V_t \rho_g / \mu$
		A. Sand		
30–40	0.534	12.1	.....	.....
	0.534	12.4	0.933	128
	0.568	12.7	0.941	139
	0.583	13.5	0.900	149
	0.594	14.6	0.875	162
	0.584	15.9	0.671	173
40–60	0.401	8.85	.....	.....
	0.427	9.35	1.36	75.8
	0.435	9.88	1.25	81.5
	0.465	10.2	1.24	90.8
	0.490	10.8	1.18	100
	0.518	12.1	0.966	120
60–80	0.313	5.50	.....	.....
	0.308	6.05	2.54	34.9
	0.319	6.57	2.12	39.3
	0.335	7.08	1.92	44.5
	0.387	7.57	1.81	52.4
		B. Carborundum		
No. 40	0.464	8.48	.....	.....
	0.490	9.00	1.52	79.8
	0.490	9.70	1.33	85.5
	0.483	10.1	1.17	89.1
	0.499	10.9	1.07	97.8
	0.523	11.5	0.988	109
	0.535	12.3	0.893	119
No. 60	0.272	5.65	.....	.....
	0.302	6.12	2.08	33.1
	0.326	7.18	1.75	40.5
	0.365	8.06	1.49	51.9
	0.381	8.79	1.27	59.9
		C. Aloxite		
No. 46	0.480	10.5	.....	.....
	0.494	11.5	1.18	101
	0.507	12.5	1.02	113
	0.524	13.7	0.880	128
	0.591	15.7	0.791	162
No. 60	0.331	7.56	.....	.....
	0.348	8.50	1.56	51.6
	0.371	9.58	1.31	62.5
	0.410	10.7	1.16	77.2
	0.430	12.3	0.941	92.2

*Based on physical properties of air at 85 deg. F.

D. Terminal Velocity and Drag Coefficient of Particles

The terminal velocity of the particles was measured by air elutriation tests using a method similar to that of Martin.* The procedure was as follows: About 25 grams of the material from each screen fraction was placed in a brass cone at the base of the elutriation tube. This was a glass tube 1 in. in diameter and 51 in. high. Air was blown into the cone through an inverted nozzle. The velocity was measured by means of a calibrated Venturi meter. Beginning at a low velocity the air velocity was gradually increased until a few particles were thrown a short distance above the top of the elutriator tube. At this velocity, the flow of air was maintained constant until only a very few particles were being ejected, and these were rising only a short distance above the tube. The air was then shut off and the material collected outside of the tube was weighed. An effort was made to adjust the air velocity so that each fraction had approximately the same weight. Several fractions were collected from each sample placed in the elutriator, and the average diameter of the particles in each fraction was determined as described in the foregoing. In calculating the drag coefficients, the arithmetic average of the air velocities of two successive sub-fractions was taken as the terminal velocity of the particle having the average diameter of the second sub-fraction. The coefficients were calculated from the equation

$$C_D = \frac{8K}{\pi} \cdot \frac{\rho_p - \rho_g}{\rho_g} \cdot \frac{D_p g}{V_t^2}. \tag{60}$$

It may be noted that in this equation the projected area of the particle on which the drag resistance is based is taken as $\pi D_p^2/4$. There may be some question as to whether this is correct, since it has already been shown that the volume-shape factor was less than $\pi/6$ for all of the materials studied. For convex surfaces, however, the error involved in computing the projected area as that of a sphere having the diameter D_p is less than that in computing the volume on this basis. Furthermore, the projected area calculated in this way agrees closely with that measured directly, as is shown in Table 4.

Values of the drag coefficient calculated for the various materials are shown in Table 5. The table also shows the variation in the size of the particles in each screen fraction used in the heat transfer

*Geoffrey Martin, Trans. Inst. Chem. Engrs., London, vol. 4, pp. 164-78, 1926.

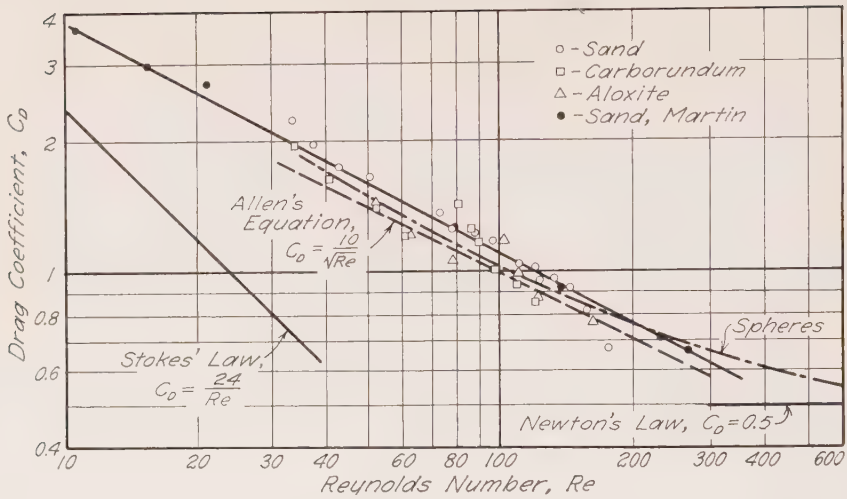

Fig. 7. Drag Coefficients for Particles Falling Through Air

measurements. The drag coefficients are also plotted in Fig. 7. The data agree excellently with Martin's results on carefully sized fractions of sand particles. When the volume-shape factors were not included, the points representing the data on the three materials were scattered considerably. Since inclusion of the factors brings the data together, the surface roughness evidently is not important in determining the frictional coefficient. The carborundum particles were considerably rougher than the sand, as shown by the smaller shape factor. Under the microscope also these particles appeared to have much more irregular shapes than the sand and aloxite particles. Martin's volume-shape factor for sand was 0.284, this indicating very rough grains. It is probable, however, that he did not take precautions to prevent preferred orientation of the particles on the microscope slide and that the lower value may be explained on this basis.

From the drag coefficients, the time of fall was calculated from the familiar equation of accelerated motion,*

$$\frac{dV}{d\theta} = g - \frac{\pi C_D}{8K} \frac{\rho_g}{\rho_p} \frac{V^2}{D_p}. \tag{61}$$

*A slight error is recognized in this calculation since the effective mass of a body is greater for accelerated motion than for uniform motion due to the inertia of the ambient fluid. The mass is increased by approximately one-half of that of the displaced fluid.

TABLE 6
CALCULATED TIMES OF FALL OF PARTICLES THROUGH FURNACES

Screen Size	Ambient Gas	Time in Heated Section of Tube, sec.	
		Small Furnace	Large Furnace
A. Sand			
30–40	Air	0.474	0.823
40–60	Air	0.518	0.976
40–60	CO_2	0.605	.....
60–80	Air	0.613	1.258
B. Carborundum			
No. 40	Air	0.511	.....
No. 60	Air	0.619	.....
C. Aloxite			
No. 46	Air	0.485	.....
No. 60	Air	0.552	.....

Equation (61) was integrated graphically, or by a series approximation in the range between Reynold's number of 10 and 200 where

$$C_D = 12.8Re^{-0.53}. \tag{62}$$

The distance of fall as a function of velocity was likewise determined by a second series integration. The time required for a particle to pass through the heated zone was then found from graphs of the velocity-time and distance-velocity functions.

It may be observed that the product $C_D\rho$ is nearly independent of temperature and, therefore, for any one gas, the time of fall calculated for one temperature may be safely used at other temperatures not too far removed. The viscosity of air varies approximately as the 0.7 power of the absolute temperature. Since the density varies as the −1.0 power, the Reynolds number varies as the −1.7, and the drag coefficient as the 0.9 power. Thus $C_D\rho_g$ must depend only on the −0.1 power of the absolute temperature. In the experimental work, the maximum variation of the "film" temperature of the particles was from approximately 300 deg. F. to 800 deg. F. This would cause a variation in $C_D\rho_g$ of only 5.5 per cent.

Table 6 shows the calculated average times of fall for the various materials studied in the two furnaces.

IV. Correlation of Experimental Data

11. *Experimental Results.*—The complete experimental data in the heat transfer measurements are given in Tables 7, 8, and 9. The runs are classified according to the furnace wall temperature, particle size, quantity of material fed, size of furnace, and nature of the ambient gas. It will be observed that the basic quantities measured by the calorimetric observations give the final particle temperature as well as the total heat transferred. The logarithmic mean temperature difference between the wall and the particle was calculated and, from this, the rate of heat transfer per degree difference was computed. On the basis of the feed rate, surface area of each particle, and the time of exposure, the overall heat transfer coefficient, designated as h_m, was then calculated. The radiation coefficient, h_r, was estimated on the basis of Equation (57). The overall convection coefficient, based on the particle area, is equal to $(h_m - h_r)$.

A. Sample Calculation

A sample calculation of the data, based on Run 95, is shown below.

Weight of feed ..... 100 gm.
Duration of feed ..... 80.3 sec.

Cross-sectional area of furnace ..... $\left(\frac{\pi}{4}\right)\left(\frac{1.61}{12}\right)^2 = 0.0141$ sq. ft.

F, feed rate ..... $\frac{100 \times 60 \times 1}{453 \times 80.3 \times 0.0141} = 11.7$ (lbs.)/(min.)(sq. ft.)

Δ, increase in particle temperature ..... 361 deg. F.
Mean specific heat of quartz 0.205

Heat transferred during run ..... $\frac{100}{453} \times 0.205 \times 3.5 = 16.6$ B.t.u.

Rate of heat transfer ..... $\frac{16.6 \times 3600}{80.3} = 745$ B.t.u./hr.

t_w, average wall temperature ..... 858 deg. F.
$(t_w - t_{p1})$, temperature difference at top of furnace ..... $858 - 75 = 783$ deg. F.
$(t_w - t_{p2})$, temperature difference at bottom of furnace ..... $858 - (75 + 361) = 422$ deg. F.

Δt_{lm}, logarithmic mean temperature difference ..... $\frac{783 - 422}{ln \frac{783}{422}} = 585$ deg. F.

γ, area ratio ..... $CDa = \frac{\theta F D a \Delta}{60 w_p L} = \frac{0.475 \times 11.7 \times 1.61 \times 2.65 \times 10^{-6}}{60 \times 4.12 \times 10^{-7} \times 48} = 0.0198$

A_w, wall area ..... $\pi DL = 1.686$ sq. ft.
NA_p, total particle area in tube ..... $\gamma A_w = 1.686 \times 0.0198 = 0.0334$ sq. ft.

h_m, overall heat transfer coefficient ..... $\frac{16.6 \times 3600}{80.3 \times 585 \times 0.0334}$
$= 38.2$ B.t.u./(hr.)(sq. ft.)(deg. F.)

ϵ_p, absorptivity of quartz*.....0.5; $(\epsilon_p CDa) = 0.5 \times 0.0198 = 0.0099$

ϵ_c, absorptivity of cloud (Table 1).....0.0099

h_r, radiation coefficient, based on arithmetic average particle temperature.....

$$\frac{0.172 \times 0.0099}{0.0198}\left[\frac{\left(\frac{858+460}{100}\right)^4 - \left(\frac{255+460}{100}\right)^4}{585}\right] = 3.6$$

$(h_m - h_r)$, net transfer coefficient due to conduction-convection exchange alone..

$38.2 - 3.6 = 34.6$ B.t.u./(hr.)(sq. ft.)(deg. F.)

B. Validity of Assumptions Made in Calculation of Data

In the calculation of the heat transfer coefficient as described in the foregoing the following basic assumptions were made: (1) that a steady state was rapidly established between the wall and the gas, (2) that the average temperature of the particle as determined from the calorimeter measurements was equal to the surface temperature and (3) that the overall rate of heat transfer is proportional to the logarithmic mean temperature difference between the wall and the particle. These assumptions will now be investigated.

The first assumption appears to be logical on account of the small heat capacity of the gas in the furnace and the high heat conductivity of the walls. As proof that a steady state between the wall and the gas was reached, several series of runs under various conditions were made at approximately constant rates of feed for varying lengths of time. These are shown in Table 7E to 7I, inclusive. When the observed heat transfer coefficients are corrected for slight variations in the feed rate, the results indicate that the steady state was reached in less than ten seconds, as no significant variation in the overall coefficient was found over periods from ten seconds to four minutes even at high rates of feed.

The second assumption requires that the unsteady radial flow of heat within the particle be rapid compared with the rate of heat transmission up to the particle surface. The ratio of the thermal resistance outside the particle to that inside is equal to $2k/h_{cp}D_p$. McAdams† shows that when this ratio is greater than about 6, the temperature gradient within the particle may be neglected. It will be shown later that the experimental values of the surface coefficient vary from about 50 B.t.u./(hr.)(sq. ft.)(deg. F.) for the largest particles to about 30 for the smallest. Using the value 4.2 B.t.u./(hr.)(sq. ft.)(deg. F. ft.) for the thermal conductivity of quartz, which has the lowest value of any of the three materials studied, the values of the thermal resistance

*K. Hild, Mitt. Kaiser Wilhelm Inst. Eisenforschg. (Dusseldorf), vol. 14, Part 200, pp. 59-70, 1932.

†W. H. McAdams, "Heat Transmission," p. 37, McGraw-Hill Book Co., New York, 1933

TABLE 7

HEAT TRANSFER TO CLOUDS OF SAND PARTICLES

Run	θ sec.	F lb./(min.) (sq. ft.)	t_w deg. F.	Δ deg. F.	Δt_{lm} deg. F.	NA_p sq. ft.	h_m	h_r	$h_m - h_r$
							B.t.u./(hr.)(sq. ft. particle surface)(deg. F.)		
A. Small Furnace; Nominal Wall Temperature 1050 deg. F.									
a. Particle size—30–40 mesh; 100 gm. fed									
136	39.6	23.7	1074	414	775	0.0694	33.0	5.7	27.3
137	37.0	25.4	1072	410	775	0.0745	32.6	5.7	26.9
138	35.3	26.6	1073	413	774	0.0780	33.0	5.7	27.3
139	38.0	24.7	1073	405	780	0.0724	32.0	5.7	26.3
140	323	2.92	1066	530	693	0.0856	48.5	6.2	42.3
141	76.7	12.4	1058	444	739	0.0364	37.1	5.7	31.4
142	86.7	10.8	1042	428	733	0.0316	36.3	5.5	30.8
143	210.8	4.45	1064	524	697	0.0130	48.0	6.1	41.9
144	254.8	3.68	1057	522	690	0.0108	48.0	6.1	41.9
145	125.8	7.46	1054	468	722	0.0218	40.7	5.8	34.9
b. Particle size—40–60 mesh; 100 gm. fed									
126	25.4	37.9	1092	436	780	0.1330	28.2	5.4	22.8
127	40.4	23.2	1075	454	750	0.0835	30.8	6.0	24.8
128	45.2	20.7	1069	463	738	0.0745	32.1	5.9	26.2
129	127.8	7.35	1072	570	672	0.0264	44.6	6.5	38.1
130	197.0	4.76	1067	600	638	0.0171	49.8	6.7	43.1
131	108.6	8.65	1062	560	669	0.0312	43.8	6.3	37.5
132	53.2	17.6	1054	475	715	0.0634	34.1	5.7	28.4
133	147.8	6.35	1045	580	636	0.0228	48.0	6.3	41.7
134	73.8	12.7	1044	509	684	0.0457	38.6	5.9	32.7
c. Particle size—60–80 mesh; 100 gm. fed									
146	184.0	5.10	1051	686	583	0.038	29.8	6.8	23.0
147	76.4	12.3	1045	593	618	0.092	24.8	6.3	18.5
148	250.2	3.75	1039	678	558	0.028	31.8	6.9	24.9
149	54.6	17.2	1027	558	633	0.128	22.3	5.9	16.4
150	25.0	37.5	1067	461	738	0.280	15.4	5.7	9.7
151	51.6	18.2	1053	547	667	0.136	20.7	6.0	14.7
152	50.4	18.6	1043	578	636	0.139	23.2	6.1	17.1
153	221.4	4.24	1033	639	580	0.032	28.4	6.6	21.8
154	122.5	7.66	1027	616	591	0.057	26.4	6.3	20.1
B. Small Furnace; Nominal Wall Temperature 850 deg. F.									
a. Particle size—30–40 mesh; 100 gm. fed									
94	34.4	27.2	865	315	620	0.0796	30.2	3.9	26.3
95	80.3	11.7	858	361	585	0.0343	38.2	3.6	34.6
96	148.0	6.34	852	385	563	0.0186	41.7	4.0	37.7
97	128.5	7.30	849	371	569	0.0214	40.8	4.0	36.8
98	226.9	4.13	846	420	533	0.0121	48.8	4.2	44.6
99	140.0	6.70	833	376	549	0.0196	41.8	3.9	37.9
100	86.0	10.9	827	371	545	0.0320	41.3	4.1	37.2
101	100.4	9.34	823	351	555	0.0274	38.0	3.8	34.2
102	82.6	11.3	816	342	551	0.0332	37.4	3.7	33.7
103	104.6	8.97	858	340	596	0.0263	34.0	4.0	30.0
105	30.4	30.8	846	302	609	0.0903	29.2	3.7	25.5
106	40.0	23.4	839	302	602	0.0685	29.6	3.7	25.9
107	28.8	32.6	835	300	598	0.0955	29.5	3.7	25.8

TABLE 7 (CONTINUED)
HEAT TRANSFER TO CLOUDS OF SAND PARTICLES

Run	θ sec.	F lb./(min.)(sq. ft.)	t_w deg. F.	Δ deg. F.	Δt_{lm} deg. F.	NA_p sq. ft.	h_m	h_r	$h_m - h_r$
							B.t.u./(hr.)(sq. ft. particle surface)(deg. F.)		
b. Particle size—40–60 mesh; 100 gm. fed									
108	32.6	28.8	839	339	581	0.1037	28.4	3.8	24.6
108b	36.4	25.8	832	340	572	0.0929	28.8	3.7	25.1
109	46.3	20.2	827	360	554	0.0727	32.0	3.8	28.2
110	121.4	7.72	824	414	514	0.0278	40.6	3.7	36.9
111	101.9	9.21	868	239	545	0.0332	40.4	4.4	36.0
112	191.0	4.91	860	468	517	0.0177	46.1	4.5	41.6
114	102.4	9.15	848	417	540	0.0330	38.8	4.2	34.6
115	85.5	10.9	843	422	548	0.0394	38.6	4.1	34.5
116	53.4	17.6	859	378	572	0.0634	32.8	4.1	28.7
c. Particle size—60–80 mesh; 100 gm. fed									
117	72.3	12.9	853	451	521	0.0961	21.2	4.2	17.0
118	76.7	12.2	842	452	518	0.0924	21.1	4.2	16.9
118b	173.6	5.40	841	505	486	0.0402	26.8	4.5	22.3
119	200.4	4.67	841	532	447	0.0348	29.9	4.7	25.2
120	150.5	6.23	853	540	455	0.0464	29.9	4.8	25.1
121	116.4	8.05	848	501	481	0.0600	26.0	4.5	21.5
122	43.8	21.4	842	428	526	0.1596	19.8	4.1	15.7
123	114.0	8.23	833	499	466	0.0614	26.6	4.5	22.1
124	61.6	15.2	824	439	498	0.1133	21.4	4.0	17.4
125	27.9	33.6	826	351	558	0.250	14.9	3.7	11.2
C. Small Furnace; Nominal Wall Temperature 700 deg. F.									
a. Particle size—30–40 mesh; 100 gm. fed									
180	359.2	2.61	723	346	455	0.0076	45.7	3.2	42.5
181	142.0	6.60	718	318	467	0.0193	40.4	3.1	37.4
182	250.0	3.75	716	332	456	0.0110	43.7	3.1	40.6
183	99.6	9.42	714	300	475	0.0276	37.3	3.1	34.2
184	62.2	15.1	715	286	485	0.0442	34.4	3.0	31.4
185	32.0	29.3	714	249	506	0.0858	28.2	2.9	25.3
186	28.6	32.8	714	246	507	0.0960	27.9	2.8	25.1
195	51.0	18.4	698	257	484	0.0539	30.6	2.8	27.8
b. Particle size—40–60 mesh; 100 gm. fed									
170	58.7	16.0	698	318	447	0.0575	35.0	3.0	32.0
171	89.8	10.5	695	340	426	0.0378	38.6	3.1	35.5
172	59.7	15.7	693	310	446	0.0565	33.4	2.9	30.5
173	66.8	14.0	693	320	438	0.0504	35.6	3.0	32.6
174	150.0	6.25	695	384	412	0.0225	43.5	3.3	40.2
175	312.7	3.0	693	382	396	0.0108	48.0	3.2	44.8
176	216.7	4.35	691	375	399	0.0157	46.5	3.2	43.3
177	112.8	8.32	697	342	429	0.0299	39.1	3.0	36.0
178	30.5	30.7	697	260	482	0.1105	25.5	2.8	22.7
179	33.8	27.8	698	280	270	0.1000	28.3	2.9	25.4

TABLE 7 (CONTINUED)
HEAT TRANSFER TO CLOUDS OF SAND PARTICLES

Run	θ sec.	F lb./(min.) (sq. ft.)	t_w deg. F.	Δ deg. F.	Δt_{lm} deg. F.	NA_p sq. ft.	h_m	h_r	$h_m - h_r$
							B.t.u./(hr.)(sq. ft. particle surface)(deg. F.)		
c. Particle size—60–80 mesh; 100 gm. fed									
187	224.0	4.21	743	460	395	0.0314	28.6	3.8	24.8
188	89.6	10.5	737	418	420	0.0783	23.9	3.4	20.4
189	222.0	4.22	732	451	389	0.0314	28.5	3.8	24.7
190	39.7	23.6	728	347	457	0.176	17.9	3.1	14.8
191	36.4	25.8	724	320	472	0.192	15.8	3.1	12.7
192	26.8	35.0	709	324	454	0.261	16.6	2.9	13.7
193	36.0	26.0	706	336	442	0.194	18.0	3.0	15.0
194	70.0	13.4	702	368	417	0.0998	21.0	3.2	17.8
D. Small Tube Furnace; Nominal Wall Temperature 575 deg. F. Particle size—40–60 mesh; 100 gm. fed									
155	28.0	33.5	586	218	392	0.1207	25.7	2.2	23.5
156	92.4	10.1	584	298	340	0.0366	42.0	2.3	39.7
157	50.4	18.6	586	260	366	0.0670	33.6	2.2	31.4
159	202.8	4.62	578	308	326	0.0166	45.5	2.3	43.2
160	77.2	12.1	578	278	346	0.0436	38.2	2.2	36.0
161	38.9	24.1	576	239	368	0.0868	30.3	2.2	28.1
162	72.3	13.0	579	274	350	0.0468	37.0	2.2	34.8
163	31.5	29.8	582	215	390	0.1073	25.2	2.1	23.1
164	70.9	13.2	576	270	350	0.0475	36.6	2.2	34.4
165	36.1	26.0	570	220	376	0.0936	27.2	2.1	25.1
166	55.8	16.8	566	251	352	0.0605	33.4	2.1	31.3
167	128.8	7.29	563	292	320	0.0262	43.6	2.2	41.4
168	89.8	10.4	563	272	334	0.0374	38.8	2.2	36.6
169	176.2	5.32	561	294	317	0.0192	44.3	2.2	42.1

ratio, $2k/h_{cp}D_p$, vary from 94 to 246. Therefore, the assumption of negligible temperature gradient through the particle is satisfactory for the particle sizes studied in this investigation.

The validity of the assumption that the overall rate of heat transfer is proportional to the logarithmic mean temperature difference between the wall and the particle for a particle falling with accelerated motion through a tube at constant wall temperature may be proven as follows:

The heat balance on a single particle requires

$$w_p c dt = h_m A_p (t_w - t_p)\, d\theta. \tag{63}$$

If t_w is constant, this equation may be integrated between limits to give

$$ln \frac{t_w - t_{p1}}{t_w - t_{p2}} = \frac{h_m A_p}{w_p c} \Delta\theta_f. \tag{64}$$

TABLE 7 (CONTINUED)
HEAT TRANSFER TO CLOUDS OF SAND PARTICLES

Run	Weight Feed gm.	θ sec.	F lb./(min.)(sq. ft.)	t_w deg. F.	Δ deg. F.	Δt_{lm} deg. F.	NA_p sq. ft.	h_m	h_r	$h_m - h_r$
								B.t.u./(hr.)(sq. ft. particle surface)(deg. F.)		
E. Large Furnace; Nominal Wall Temperature 1000 deg. F.										
a. Particle size—30–40 mesh; varying amounts fed as indicated										
13	448	30.0	25.8	992	492	614	0.712	29.4	5.5	23.9
14	378	45.0	14.5	992	559	567	0.400	36.8	5.8	31.0
15	434	90.0	8.32	981	600	524	0.230	43.2	6.1	37.1
16	334	120.0	4.80	973	632	492	0.1327	49.0	6.1	42.9
17	443	25.0	30.5	984	467	622	0.842	27.4	5.4	22.0
18	342	180.0	3.28	997	672	486	0.445	37.1	5.9	31.2
b. Particle size—40–60 mesh; varying amounts fed as indicated										
20	452	25.0	31.2	985	508	596	0.1155	23.4	5.5	17.9
21	406	45.0	15.6	995	623	522	0.577	33.8	6.3	27.5
22	328	180.0	3.14	998	745	420	0.1161	52.3	7.7	44.6
23	356	25.0	24.6	983	547	566	0.910	26.8	5.7	21.1
24	401	50.0	13.8	967	625	490	0.511	36.2	6.1	30.1
25	278	60.0	8.00	977	700	437	0.296	46.4	6.9	39.5
26	233	90.0	4.46	984	746	402	0.1652	54.2	7.7	46.5
F. Small Furnace; Nominal Wall Temperature 1050 deg. F.										
a. Particle size—30–40 mesh; varying amounts fed as indicated										
196	18.8	30.0	5.88	1050	484	706	0.0168	44.4	5.7	38.8
197	28.5	60.0	4.45	1045	533	670	0.0127	52.5	5.8	46.7
198	75.5	120.0	5.90	1044	508	685	0.0169	48.1	5.7	42.4
199	102.8	180.0	5.35	1041	513	680	0.0153	49.2	5.7	43.5
200	180.7	240.0	7.05	1039	489	692	0.0202	45.7	5.6	40.1
201	41.2	30.0	12.9	1045	452	722	0.0369	40.0	5.6	34.4
202	81.6	60.0	12.8	1043	456	720	0.0366	40.4	5.6	34.9
203	116.0	90.0	12.1	1042	460	714	0.0346	41.3	5.5	35.8
204	165.9	120.0	13.0	1037	448	715	0.0372	39.8	5.4	34.4
205	242.7	180.0	12.7	1031	448	719	0.0363	39.7	5.5	34.2
b. Particle size—40–60 mesh; varying amounts fed as indicated										
207	15.1	15	9.44	1035	553	644	0.0330	47.6	5.8	41.8
208	31.2	30	9.76	1035	550	647	0.0341	46.0	5.8	40.2
209	43.9	45	9.15	1034	562	638	0.0320	47.8	5.8	42.0
210	68.5	75	8.56	1031	548	644	0.0300	45.7	5.7	40.0
G. Small Furnace; Nominal Wall Temperature 725 deg. F. Particle size—40–60 mesh; varying amounts fed as indicated										
228	255	60.0	39.9	718	268	502	0.140	26.1	2.8	23.3
229	152	40.0	35.6	717	279	490	0.125	27.8	2.8	25.0
230	84.5	20.2	39.2	719	270	500	0.137	26.6	2.9	23.7
231	42.7	10.0	40.1	719	274	495	0.140	27.0	2.9	24.1
232	48.9	120.0	3.82	721	395	418	0.0134	48.1	3.2	44.9
233	90.5	240.0	3.54	721	408	409	0.0124	51.3	3.2	48.1
234	8.9	30.0	2.78	722	411	407	0.0097	52.1	3.2	48.9
235	37.4	100.0	3.51	721	409	408	0.0123	51.6	3.2	48.4

TABLE 7 (CONCLUDED)
HEAT TRANSFER TO CLOUDS OF SAND PARTICLES

Run	Weight Feed gm.	θ sec.	F lb./(min.)(sq. ft.)	t_w deg. F.	Δ deg. F.	Δt_{lm} deg. F.	NA_p sq. ft.	h_m	h_r	$h_m - h_r$
								B.t.u./(hr.)(sq. ft. particle surface)(deg. F.)		
H. Small Furnace; Nominal Wall Temperature 1050 deg. F. Particle size—40–60 mesh; varying amount fed as indicated; atmosphere of CO_2										
211	89.9	75	11.3	1035	576	629	0.0427	45.9	6.2	39.7
212	35.9	45	7.48	1038	592	621	0.0282	48.2	6.5	41.7
213	9.5	15	5.94	1041	649	582	0.0224	57.5	6.7	50.8
214	21.3	30	6.65	1042	637	593	0.0251	54.9	6.7	48.2
215	94.4	120	7.38	1034	617	599	0.0279	52.3	6.5	45.8
I. Small Furnace; Nominal Wall Temperature 725 deg. F. Particle size—40–60 mesh; varying amounts fed as indicated; atmosphere of CO_2										
224	39.6	10	37.8	732	292	497	0.1407	26.8	3.1	23.7
225	76.1	20	36.3	730	302	490	0.1350	26.8	2.9	23.9
226	164.4	40	39.2	727	288	495	0.1460	26.4	3.0	23.4
227	240.3	60	38.2	723	283	495	0.1421	26.0	3.0	23.0

Here $\Delta\theta_f$ is the time of fall. The true average temperature difference is seen to be the logarithmic mean if the expression for the total heat transferred in time is written

$$w_p c\,(t_{p2} - t_{p1}) = h_m A_p\,(t_w - t_p)_{\text{av.}} \Delta\theta_f. \tag{65}$$

By comparison with Equation (64) it is seen that

$$(t_w - t_p)_{\text{av.}} = \frac{t_{p2} - t_{p1}}{ln\dfrac{t_w - t_{p1}}{t_w - t_{p2}}} = (t_w - t_p)_{\text{lm.}} \tag{66}$$

The derivation of Equation (64) is based on the assumption of a constant value of h_m. Numerical integration of Equation (63) using for h_m theoretical values obtained from individual coefficients calculated by means of Equations (36), (57), and (58) showed that under typical conditions h_m might increase from 30 to 36 B.t.u./(hr.)(sq. ft.)(deg. F.), and remain constant at the latter value over the lower half of the furnace. This change was too small to affect the validity of the logarithmic mean assumption.

TABLE 8
HEAT TRANSFER TO CLOUDS OF CARBORUNDUM PARTICLES

Run	θ sec.	F lb./(min.) (sq. ft.)	t_w deg. F.	Δ deg. F.	Δt_{lm} deg. F.	NA_p sq. ft.	h_m	h_r	$h_m - h_r$
							B.t.u./(hr.)(sq. ft. particle surface)(deg. F.)		
A. Small Furnace; Nominal Wall Temperature 1050 deg. F.									
a. Particle size—No. 40; 100 gm. fed									
C19	496.0	1.89	1052	627	611	0.0069	55.1	6.8	48.3
C20	193.0	4.86	1046	577	641	0.0179	47.4	6.4	41.0
C21	116.2	8.09	1039	539	658	0.0296	42.9	6.2	36.7
C22	72.2	13.0	1038	506	674	0.0475	39.0	5.9	33.1
C23	58.2	16.1	1032	498	676	0.0593	38.0	5.8	32.2
C24	38.0	24.7	1020	452	692	0.0905	33.2	5.5	27.7
C25	49.2	19.0	1017	473	675	0.0700	35.8	5.6	30.2
C26	29.0	32.4	1020	429	706	0.1185	30.6	5.4	25.2
b. Particle size—No. 60; 100 gm. fed									
C27	286.0	3.28	1047	697	553	0.0238	33.4	7.2	26.2
C28	146.4	6.40	1035	662	565	0.0465	32.0	6.9	25.1
C29	118.0	7.95	1028	641	578	0.0576	30.2	6.6	23.6
C30	78.0	12.0	1022	613	590	0.0870	28.3	6.3	22.0
C31	59.2	15.8	1021	583	586	0.1146	25.5	6.1	19.4
C32	42.0	22.3	1022	548	635	0.1618	22.7	6.0	16.7
C33	34.9	28.9	1026	543	643	0.1950	22.4	5.9	16.5
C34	27.9	33.6	1028	512	666	0.244	20.2	5.7	14.5
B. Small Tube Furnace; Nominal Wall Temperature 725 deg. F.									
a. Particle size—No. 40; 100 gm. fed									
C1	45.2	20.8	720	290	475	0.0765	26.8	3.0	23.8
C2	21.8	43.0	714	283	495	0.1583	23.5	2.8	20.7
C3	27.3	24.4	707	289	486	0.1267	24.8	2.9	21.9
C4	39.7	23.6	705	282	475	0.0869	26.6	2.9	23.7
C5	273.0	3.44	709	375	420	0.0127	44.0	3.3	36.7
C6	91.0	15.4	705	317	454	0.0566	31.9	3.0	28.9
C7	93.7	10.0	745	364	464	0.0368	35.0	3.3	31.7
C8	180.6	5.19	743	393	443	0.0191	41.9	3.5	38.4
C9	155.0	6.05	738	385	442	0.0222	41.5	3.5	38.0
b. Particle size—No. 60; 100 gm. fed									
C10	207.8	4.52	732	463	380	0.0328	29.2	3.7	25.5
C11	375.7	2.50	728	471	368	0.0181	30.4	3.7	26.7
C12	124.8	7.52	725	434	394	0.0545	26.4	3.5	22.9
C13	97.3	9.65	721	415	403	0.0700	24.5	3.4	21.1
C14	70 2	13.4	712	385	414	0.0970	22.1	3.2	18.9
C15	46.2	20.3	712	358	434	0.1472	18.5	3.1	15.4
C16	26.4	33.0	710	320	455	0.239	16.4	2.9	13.5
C17	20.8	45.1	707	315	488	0.327	15.1	2.8	12.3
C18	39.6	24.3	707	343	440	0.1762	18.4	3.0	15.4

12. *Analysis of Data.*—When a cloud of particles falls through a heated gas space under the influence of gravity, the time of contact of a single particle with the hot gas is essentially independent of the rate of feed. On the other hand, the ratio of the area of the particles to the area of the wall, γ, should be directly proportional to the feed rate.

TABLE 9
HEAT TRANSFER TO CLOUDS OF ALOXITE PARTICLES

Run	θ sec.	F lb./(min.)(sq. ft.)	t_w deg. F.	Δ deg. F.	Δt_{lm} deg. F.	NA_p sq. ft.	h_m	h_r	$h_m - h_r$
							B.t.u./(hr.)(sq. ft. particle surface)(deg. F.)		
A. Small Furnace; Nominal Wall Temperature 1050 deg. F.									
a. Particle size—No. 46; 100 gm. fed									
A16	395.0	2.38	1032	563	634	0.0076	52.5	6.1	46.4
A17	187.2	5.62	1031	525	660	0.0179	47.5	5.9	41.6
A18	92.6	10.1	1027	454	702	0.0323	37.8	5.6	31.2
A19	214.8	4.36	1073	550	692	0.0140	50.5	6.5	44.0
A20	151.0	6.21	1072	537	693	0.0199	48.3	6.4	41.9
A21	103.8	9.05	1070	508	713	0.0290	44.1	6.2	37.9
A22	68.8	13.6	1070	467	740	0.0435	39.3	6.0	33.3
A23	46.3	20.2	1072	425	769	0.0646	34.0	5.8	28.2
A24	36.2	25.9	1070	400	780	0.0829	31.1	5.7	25.4
A25	30.2	31.0	1062	380	773	0.0992	29.4	5.5	23.9
A26	24.2	38.8	1082	356	800	0.1240	26.4	5.4	21.0
b. Particle size—No. 60; 100 gm. fed									
A27	197.2	4.75	1043	613	613	0.0220	44.0	6.6	37.4
A28	252.0	3.72	1038	631	593	0.0172	48.4	6.9	41.5
A29	90.6	10.3	1069	555	681	0.0524	32.6	6.4	26.2
A30	54.2	17.3	1062	508	704	0.0799	31.2	6.1	25.1
A31	64.0	14.6	1055	527	685	0.0679	33.2	6.1	27.1
A32	35.6	26.3	1051	453	726	0.1214	31.8	6.1	25.7
A33	31.6	29.7	1043	425	737	0.1371	29.3	5.8	23.5
B. Small Furnace; Nominal Wall Temperature 700 deg. F.									
a. Particle size—No. 46; 100 gm. fed									
A8	289.2	3.24	716	340	450	0.0104	43.5	3.2	40.3
A9	111.6	8.41	714	306	470	0.0269	37.2	3.1	34.1
A10	184.0	5.10	713	320	461	0.0163	40.6	3.2	37.4
A11	67.4	13.9	712	280	483	0.0445	32.8	3.0	29.8
A12	32.8	28.6	711	230	513	0.0915	27.6	2.8	24.8
A13	43.6	21.4	711	247	502	0.0688	27.8	2.9	24.9
A14	21.5	43.6	712	205	527	0.1396	21.4	2.7	18.7
A15	17.6	53.3	715	196	530	0.1704	20.5	2.7	17.8
b. Particle size—No. 60; 100 gm. fed									
A1	178.0	5.26	711	382	418	0.0244	36.6	3.3	33.3
A2	225.6	4.15	704	393	403	0.0192	39.4	3.3	36.1
A3	74.0	12.7	698	334	435	0.0582	30.4	3.0	27.4
A4	103.2	9.08	692	354	417	0.0420	33.4	3.1	30.3
A5	29.2	31.1	687	270	465	0.1483	22.8	2.8	20.0
A6	36.8	25.5	722	288	490	0.1179	23.1	3.0	20.1
A7	51.0	18.8	718	313	470	0.0869	25.2	3.0	22.2

Now if we may assume as an approximation that the individual coefficients, h_{cp} and h_{cw}, are essentially independent of the feed rate, it follows that the ratio of the convective resistance at the wall to that at the particle, $NA_p h_{cp}/A_w h_{cw}$, must likewise be proportional to the feed rate. As shown in Tables 7, 8, and 9 for each series of runs the feed rate was varied over a range of nearly fifteen-fold while other

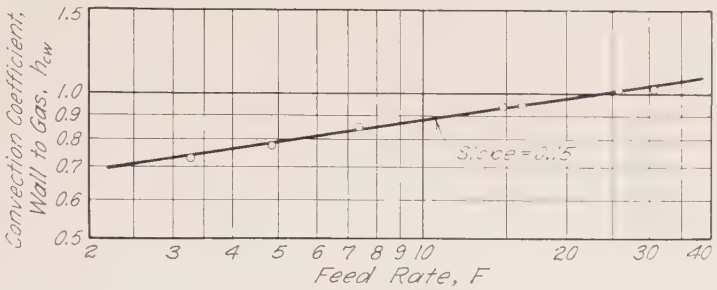

FIG. 8. CALCULATED EFFECT OF FEED RATE ON CONVECTION COEFFICIENT FROM WALL TO GAS

conditions were maintained constant. The corresponding variation in the resistance ratio thus permits estimation of the individual convection coefficients. For this purpose either of the following equations might be used:

$$\frac{1}{h_m - h_r} = \frac{1}{h_{cp}} + \frac{\gamma}{h_{cw}} \tag{67}$$

$$\frac{1}{\gamma\,(h_m - h_r)} = \frac{1}{h_{cw}} + \frac{1}{\gamma h_{cp}}. \tag{68}$$

For the first equation a plot of $1/(h_m - h_r)$ against F is required, from which the reciprocals of h_{cp} and h_{cw}, respectively, may be determined from the slope and the intercept at $F = 0$. For the second equation, $1/\gamma(h_m - h_r)$ is plotted against $1/F$. The slope of the line now gives $1/h_{cp}$, and extrapolation to the zero abscissa, representing infinite feed rate, gives $1/h_{cw}$. It should be noted that in this equation $\gamma(h_m - h_r)$ represents the overall convection coefficient based on the wall area. Consequently, any error in computing the time of fall and area of the particles will not affect the value of h_{cw} obtained by the extrapolation. Possibly for this reason, and also because the assumption of the independence of the coefficients of feed rate apparently holds better at high feed rates, the second method of plotting the data was more satisfactory than the first. Before examining the data it is well to consider the validity of the foregoing assumption as far as possible from the theoretical standpoint.

According to Equation (36), which is valid for the range of Reynolds numbers covered in the experimental work, h_{cp} is affected by the feed rate indirectly through the temperature of the gas, since for

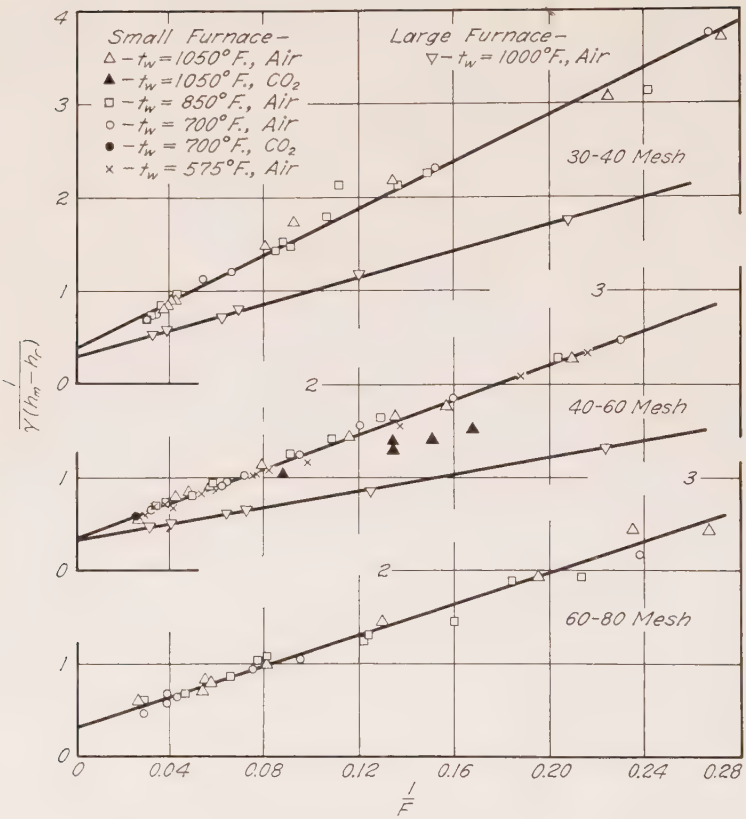

Fig. 9. Graphical Representation of Heat Transfer Data for Sand Particles

any wall temperature the gas temperature is decreased as the feed rate is increased. The effect, however, is negligible since, for air, the group $\sqrt{kV\rho c}\ D_p$ varies only about one per cent for each one hundred degrees Fahrenheit.

In the case of h_{cw} the effect of the feed rate is not so obvious. If only natural convection from the wall is considered, for a constant wall temperature, the Grashof group is expected to increase as the feed rate is increased. The approximate extent of this effect is shown in Fig. 8 in which the calculated values of the coefficient are plotted against the feed rate. For the conditions chosen the relationship is seen to be nearly exponential. The slope of the logarithmic line, however, is only 0.15, so that, from this standpoint, the approximation is nearly true.

The graphical representation of the complete data according to

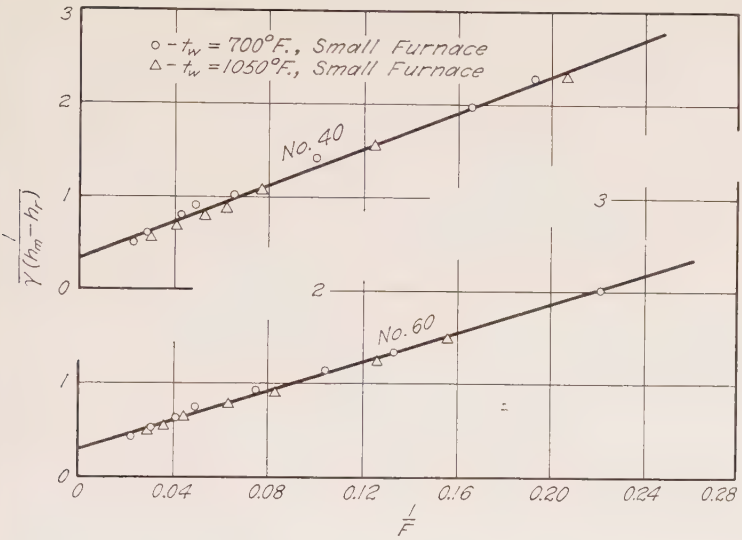

FIG. 10. GRAPHICAL REPRESENTATION OF HEAT TRANSFER DATA FOR CARBORUNDUM PARTICLES

Equation (68) is shown in Figs. 9, 10, and 11. It will be observed that, for a given furnace, the points fall very near a single straight line regardless of the wall temperature. The change in slope from one furnace to another is due principally to the change in the ratio between γ and F. Using the method of Least Squares for the linear equation the coefficients, h_{cw} and h_{cp}, were calculated for each of the series of runs. The results are shown in Table 10. It is apparent that neither of the coefficients varies with the wall temperature beyond the experimental error. Consequently, average values are justified for each particle size.

In Table 10 the values of h_{cw} and h_{cp} calculated from Equations (58) and (35), respectively, are also given. The observed values of h_{cw} range from 2.1 to 3.9 B.t.u./(hr.) (sq. ft) (deg. F.) for the small furnace and from 3.8 to 4.1 for the large furnace. The average coefficient of natural convection calculated from the Rice equation is about 1.0, and is independent of the diameter of the furnace. Undoubtedly this discrepancy is due to the increased convection caused by the particles themselves, and by the cooling of the gas at the top of the furnace. In a closed furnace there must be a circulation of the gas up the walls and down the center of the furnace. The convection at the walls apparently increases with the size of the furnace. Unfortunately, two

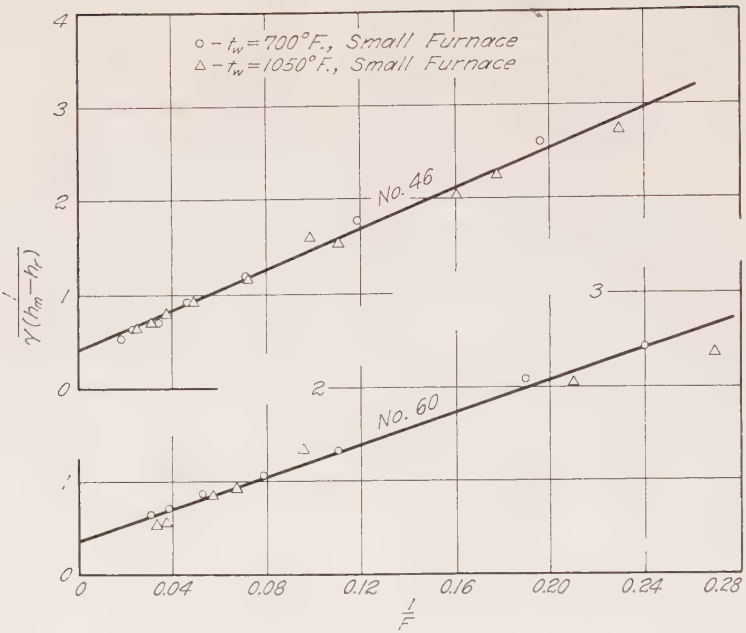

Fig. 11. Graphical Representation of Heat Transfer Data for Aloxite Particles

furnace sizes are not sufficient to establish even an empirical relationship for this effect. For values of the Grashof group between 10^4 and $10^$, a rough approximation might be made by multiplying the coefficient for natural convection by a factor equal to the diameter of the furnace in inches. For values of the group greater than 10^8 it is probably better to omit the correction factor entirely, and the overall coefficient would then be conservative. The latter range includes most cases of practical importance for which the furnace is large and heat transfer by radiation predominates, so that even large errors in the calculated value of h_{cw} can be neglected. Furthermore, in many furnaces heat is not transmitted through the walls but is introduced directly in hot gases so that h_{cw} does not enter into the calculations.

The observed values of h_{cp} agree within 20 per cent with those calculated from Equation (35) for all cases except for the smallest particles where they are about 50 per cent low. Since the observed values depend directly on the calculated time of fall, a possible explanation of the discrepancy lies in the downward motion of the gases at the center of the tube which would result in a shorter time of contact

TABLE 10
SUMMARY OF RESULTS ON INDIVIDUAL HEAT TRANSFER COEFFICIENTS

Material	Screen Size	Wall Temperature deg. F.	h_{cw}, B.t.u. (hr.)(sq. ft. wall area)(deg. F.)		h_{cp}, B.t.u. (hr.)(sq. ft. particle area)(deg. F.)	
			Observed	Calculated (Equation (58))	Observed	Calculated (Equation (35))
A. Small Furnace						
Sand	30–40	1050	2.4	0.64 to 0.98	48	54
		850	2.3	0.70 to 1.02	48	53
		700	2.3	0.63 to 1.01	47	53
		Average	2.3		48	..
Sand	40–60	1050	2.5	0.75 to 1.04	52	59
		850	2.8	0.74 to 1.02	49	59
		700	2.8	0.69 to 1.05	55	58
		575	3.1	0.75 to 1.04	52	58
		Average	2.8		52	..
Sand	60–80	1050	2.9	0.72 to 1.10	30	67
		850	2.6	0.76 to 1.05	34	66
		700	3.0	0.73 to 1.05	32	65
		Average	2.8		32	..
Carborundum	No. 40	1050	3.5	0.61 to 1.02	44	55
		700	3.0	0.72 to 1.08	46	54
		Average	3.2		45	..
Carborundum	No. 60	1050	3.9	0.69 to 1.02	30	65
		700	3.0	0.65 to 1.04	27	63
		Average	3.4		28	..
Aloxite	No. 46	1050	2.1	0.63 to 1.05	53	57
		700	2.5	0.68 to 1.10	45	56
		Average	2.3		49	..
Aloxite	No. 60	1050	2.4	0.72 to 1.03	47	64
		700	3.2	0.72 to 1.06	51	62
		Average	2.8		49	..
B. Large Furnace						
Sand	30–40	1000	4.1	0.73 to 1.03	49	54
Sand	40–60	1000	3.8	0.75 to 1.04	52	54

than that calculated. The error would be greater for the smaller particles than for the larger ones. It is unlikely that an error in the measurement of the particle areas would be positive, thus resulting in small coefficients, since concave surfaces are known to exist in all of the particles to some extent.

The values of h_m for carbon dioxide at a wall temperature of 1050 deg. F. are definitely greater than those obtained for air, as shown in Fig. 9, but the values at 700 deg. F. are essentially the same for both gases. If it is assumed that the radiation interchange between the

gas and the suspended particles is negligible, and therefore that h_{cp} is the same for the two gases, the coefficient of heat transfer for radiation from the furnace wall to the carbon dioxide can be estimated by means of the usual equations for gas radiation. The values obtained in this way for the two temperatures are, respectively, 1.0 and 0.25 B.t.u./(hr.) (sq. ft. wall area) (deg. F.). The higher value would represent an increase of about 40 per cent in the value of h_{cw} obtained without superimposed radiation. While the data are insufficient to determine the exact values of the individual coefficients by plotting, it appears that the larger values of h_m obtained when carbon dioxide is the ambient gas may be explained on the basis of the radiant interchange between the wall and the gas.

V. Conclusions

13. *Summarized Conclusions.*—The foregoing discussion may be summarized as follows:

(1) The rate of heat transmission from a hot furnace wall to clouds of falling particles is rapid. When the diameter of the furnace is small, the transmission takes place principally by convection from the wall to the gas and from the gas to the particles. The fraction transmitted by radiation increases rapidly with the size of the furnace and the temperature of the wall (*q. infra*). The factor producing the most important effect on the overall transmission is the rate at which the particles are fed to the furnace. When radiation is small, the main effect of varying the feed rate is to change the ratio of particle area to wall area, thus changing the relative magnitudes of the thermal resistances at the wall and at the particle without altering appreciably the respective convection coefficients themselves.

(2) The coefficient of heat transfer by convection from the gas to the particles is a function of the particle size and the rate of fall, and is nearly independent of the temperature of the ambient gas. This coefficient may be calculated from fundamental considerations of heat transfer to a submerged body in a moving fluid. The general equation is

$$h_{cp} = \frac{k}{D_p} \sum_{i=0}^{\infty} \frac{1 + e^{-\pi b_i^2}}{1 + b_i^4} \tag{35}$$

where the constants b_i are defined by the relationship: $2b_i^2 Re \cdot Pr$ — the squares of the roots of the first order Bessel function. For particles

having a specific gravity of about 3, and larger than 400 microns, falling through air, the foregoing equation may be simplified to

$$h_{cp} = 0.714 \sqrt{\frac{kV\rho c}{D_p}}. \tag{36}$$

(3) The coefficient of heat transfer by convection from the furnace wall to the gas is several times larger than the values predicted for natural convection alone, and is apparently a function of the size of the furnace. For furnaces less than eight inches in diameter and at wall temperatures less than 1200 deg. F., it is recommended that the natural convection coefficient be multiplied by a constant equal to the diameter of the furnace in inches. For larger furnaces, or for higher wall temperatures, the heat transmission takes place primarily by radiation, and the correction may be neglected.

(4) The radiation coefficient is a function of the concentration of particles in the cloud and of the shape of the furnace. The coefficient may be calculated from the equation

$$h_r = \frac{0.172\epsilon_c \left[(T_w/100)^4 - (T_p/100)^4\right]}{\gamma (T_w - T_p)}. \tag{57}$$

Values of the absorptivity of the cloud, ϵ_c, are given in Table 1 for spherical, cylindrical, and rectangular furnaces.

14. *Design of Large Furnace.*—In order to illustrate the application of the foregoing equations an example will be taken from the previous publication.* The problem is the design of a flash calciner for decomposing 83.2 tons of zinc sulphite hydrate per day into zinc oxide, water vapor, and sulphur dioxide. The conditions are as follows:

Feed, dry $ZnSO_3 \cdot 2\frac{1}{2}H_2O$ enters at 70 deg. F.
Particle size, 200 microns, found by experiment to be the arithmetic mean of particles just passing 100 mesh standard screen.
Dehydration temperature, 200 deg. F.
Decomposition temperature, 500 deg. F.
Specific heat of feed, 0.24 B.t.u./(lb.) (deg. F.).
Heat of dehydration, 40 800 B.t.u./lb. mole.
Heat of decomposition, 58 100 B.t.u./lb. mole.
Emissivity of particles, 0.5.

*Univ. of Ill. Eng. Exp. Sta. Bul. 324.

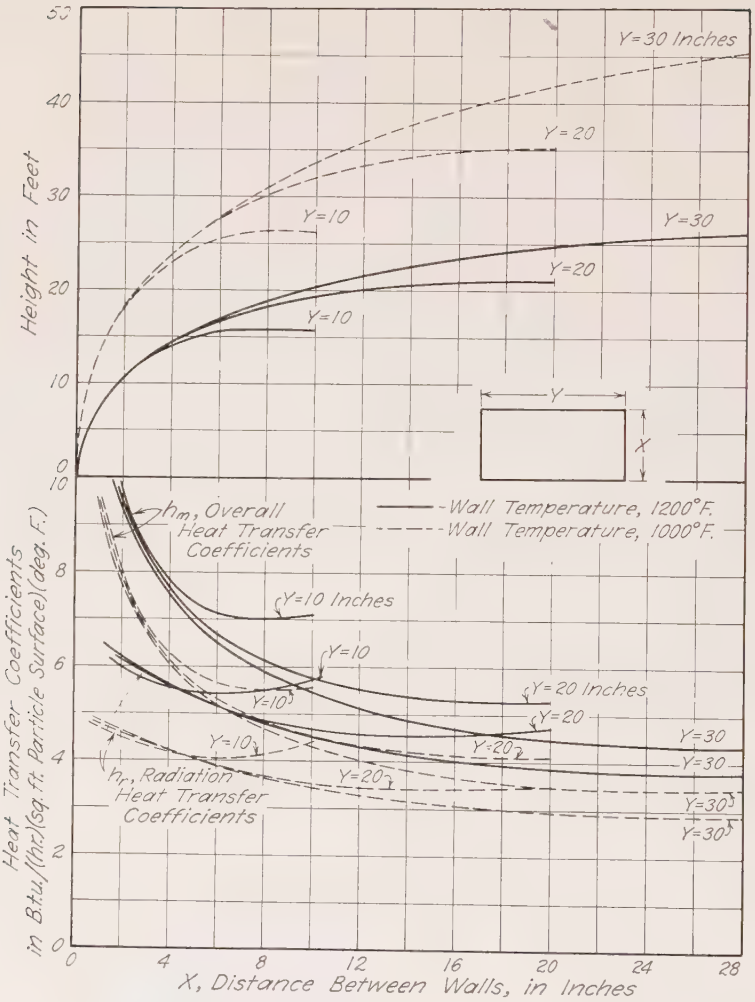

Fig. 12. Heat Transfer Coefficients and Calculated Heights of Rectangular Furnaces for Calcination of Zinc Sulphite

A feed rate of 11.7 lbs. per min. per sq. ft. is assumed so that the total cross-sectional area of the furnace will be 10 sq. ft. The furnace is to be constructed in the form of rectangular retorts with walls of high heat conductivity. Silicon carbide tile are available, 1½ inch thick, tongue and groove, with a heat conductivity of 105.5 B.t.u./(hr.) (sq. ft.) (deg. F. in.) at 1110 deg. F. and 112.5 B.t.u./(hr.) (sq. ft.) (deg. F. in.) at 2900 deg. F. The retorts may be placed in a row with

the common walls formed of slotted tile. The walls will be sealed by compression with suitably spaced springs. The furnace will be heated by horizontal gas muffles along each side, the flames traveling in several passes from bottom to top. The outside walls will be constructed of fire-brick and will be insulated.

Calculations were made to determine the effect of the shape and size of the retorts on the height required for a heat input sufficient to completely dehydrate and decompose the particles. These calculations were made by applying Equations (5), (57), and (58) to successive short sections, taking into consideration the change in the velocity, mass, and temperature of the particles and of the gas. The temperature of the furnace wall was considered to be constant. The evolved gases were assumed to leave at the bottom of the furnace so that there was an increase in the mass velocity of the gases in the two reaction zones. The reactions were considered to take place only when the particles reached the temperatures at which the respective decomposition pressures are one atmosphere, and at these temperatures the rate of reaction depended only on the rate of heat input.

The results of these calculations are plotted in Fig. 12. The following conclusions may be drawn from this study.

(1) With a uniform wall temperature of 1000 deg. F. the height of the calciner of rectangular cross section 30 inches long and 12 to 20 inches wide approaches 40 feet. Nothing is to be gained by using smaller cross sections, because the length does not decrease as rapidly as the area of the cross section decreases and, therefore, the increased number of units required more than offsets the decrease in height.

(2) An increase in wall temperature to 1200 deg. F. decreases the height by nearly 50 per cent.

(3) In these large rectangular furnaces the heat transfer is ninety per cent by radiation. If the source of heat is located at the bottom, a greater heat flux can be obtained than is shown by the curves, because of the great effect of the temperature level on radiant heat transfer. Furthermore, since the greatest part of the heat is required at the bottom in the decomposition zone, upward flow of the combustion gases is highly desirable.

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